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EVCLIDIS ELEMENTORVM LIBRI XV.

ACCESSIT LIBER XVI. DE QVINQVE SOLIDORVM REGVLARIVM inter ſe comparatione.

AD EXEMPLARIA R. P. CHRISTOphori Clauij è Societ. IESV, & aliorum collati, emendati & aucti.

COLONIAE

Apud Goſuinum Cholinum

M. D. CVII.

Cum Gratia & Priuilegio.

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MATHESEOS ET GEOMETRIAE DIVISIO.

Mathematicæ diſciplinæ, quæ omnes circa quantitatem verſantur, nomen acceperunt à Græca dictione μάθημα, ſeu μάθησις, quæ diſciplinam, & doctrinam ſignificat; eò quod tum gradatim aſcendendo doceantur, & addiſcantur; tùm ſolæ ſemper ex præcognitis quibuſdam, conceſſis, & probatis, principijs, (Haud ex hypotheſibus nondum explicatis) ad cōcluſiones demonſtrandas, quod proprium eſt doctrinarum & diſciplinarum officium, teſte Ariſtotele lib. 1. Poſter. procedant.

Pythagoras, & Mathematici vniuerſas Mathematicas diſciplinas in quatuor partes principes diſtribuũt, nempe in Arithmeticam, Muſicam, Geometriam, & Aſtronomiam. Cum enim omnis quantitas, circa quam hæ diſciplinæ verſantur, ſit vel diſcreta, ſub qua omnes numeri continentur, vel continua, ſub qua omnes magnitudines comprehenduntur; & vtraque tam ſecun-

dum ſe, quam comparatione alterius poſſit conſiderari; Viſum eſt illis has quatuor partes inſtituere, quæ vtramque quantitatem pro duplici conſideratione, diligenter contemplarentur. Itaque Arithmetica agit de quantitate diſcreta ſecundum ſe; omneſque numerorum proprietates ac paſſiones inquirit, & accuratè explicat. Muſica tractat eandẽ quantitatẽ diſcretã, ſeu numerum cum alio comparatum; quatenus ſonorum Harmoniam, & concentum reſpicit. Geometria de magnitudine, ſeu quantitate continua, ſecundum ſe quoque, vt immobilis exiſtit, diſputat. Aſtronomia denique magnitudinẽ, vt eſt mobilis, cõſiderat; qualia sũt corpora cœleſtia cõtinuè mota. Ad has aũt quatuor ſcientias Mathematicas, tum puras, tũ mixtas, omnes alię de quantitate tractan- vti Perſpectiua Geographia Etereomatria, & cæteræ, facili negocio, tanquã ad ſua capita, & fontes, ex quib. emanant, reducũtur.

Geometria apud Euclidem diuiditur in planorum (ſuperficierum planarum.) contemplationem, ſeu Geometriam proprie dictam; quæ libris ſex primis abſoluitur; Et in ſolidorum (corporum ſolidorum:) ſpeculationem, ſeu Etereomatriam; quæ libris poſtremis pertractatur. Prior quidem pars, nempe Geometria, in tres partes ſubdiuiditur. Nam in prioribus quatuor libris agitur de planis abſolutè, ita vt eorum æquali-

tas, & inæqualitas inueſtigetur. In quinto verò libro de rationalibus magnitudinum proportionibus in genere diſputatur. In ſexto denique libro proportiones figurarum planarum diſcutiuntur. Poſterior verò pars, nempe Stereometrica, in tres quoque partes ſubdiuitur. In quarum prima, videlicet libris tribus, ſeptimo, octauo, & nono, agitur de numerorum proprietatibus, paſſionibuſque ad linearum (& aliarum magnitudinum:) ſymetrarum ſeu commenſurabilium, & aſymetrarum, ſeu incommenſurabilium tractationem neceſſarijs. In ſecunda, nimirum libro decimo, ſatis prolixo, de lineis commenſurabilibus, & incommenſurabilibus, ſine quarum notitia, corpora illa quinque ſolida, regularia, ſeu platonica perfectè tractari nequeunt, diſſeritur. In tertia denique, nempe libris ſex poſtremis (qui & ipſi ſubdiuidi poterũt) de ſolidis illis acutiſſimè diſputatur, eorumque proprietates inueſtigantur. De punctis autem, & lineis in hoc opere nulla ex profeſſo extat contemplatio: quoniam Geometria potiſſimum circa figuras, quibus plana, & ſolida duntaxat (non puncta, & lineæ:) afficiuntur, verſatur.

Demonſtratio omnis Mathematicorum ab antiquis ſcriptoribus diuiditur in Problema, & Theorema. Problema quidem

vocatur ea demonſtratio, quæ iubet atque docet aliquid conſtituere, facere, inuenire, deſcribere, &c. vt ſuper data linei recta terminata triangulum æquilaterum conſtituere. Theorema verò appellatur ea demonſtratio, quæ ſolum aliquam proprietatem, ſeu paſſionem vnius, vel plurium ſimul quantitatum (multarum vel magnarum iam inuentarum:) perſcrutatur, vt in omni triangulo tres angulos eſſe æquales duobus rectis. Cæterùm tàm problema, quàm Theorema dicitur apud Mathematicos propoſitio; eò quod vtrumque aliquid nobis proponit, vti in exemplis adductis conſtat. Demonſtrationes problematum concluduntur his ferè verbis; Quod faciendum erat: Theorematum verò Demonſtrationes, his verbis; Quod oſtendendum, vel demonſtrandum erat; habitâ nimirum ratione vtriuſque. Adhæc problemata per modum Infinitum, ſed Theoremata per modum finitum ferè proferuntur. Lemma (ſumptio, vel aſſumptũ latinè) appellatur ea minus principalis, & aliqua tantum declaratione indigens, demonſtratio, quæ ad demonſtrationem alicuius problematis, vel Theorematis principalis aſſumitur, vt illa demonſtratio expeditior, ac breuior fiat; vt videre licet lib. 6 propoſ. 22. & paſſim Corollarium, ſeu poriſma, denique

que eſt, quod protenus ex facta demonſtratione tanquam lucrum aliquod additum, ſeu cōſectarium accipitur; vt videre eſt lib. 2. propoſ. 4. & paſſim.

Cùm omnis autem doctrina, omniſque diſciplina ex præexiſtente cognitione gignatur, atq; ex aſſumptis, & conceſſis quibuſdam principijs ſuas demonſtrat concluſiones: Nulla autem ſcientia, teſte Ariſtotele, ſua principia demonſtret; habebunt & Mathematicæ diſciplinæ ſua principia, ex quibus poſitis, & conceſſis ſua problemata, & Theoremata confirment. Horum autem tria ſolum genera apud Mathematicos reperiuntur. Quorum primum genus continet definitiones, quas nonnulli Hypotheſes appellant, vt punctum eſt, cuius pars nulla eſt. Secundum genus complectitur petitiones, ſeu poſtulata, quæ per ſe adeò perſpicua ſunt, vt nulla confirmatione indigeant, ſed auditoris duntaxat aſſenſum expoſcant, vt poſtuletur, vt à quouis puncto in quoduis punctum, rectam lineam ducere concedatur. Tertium denique genus comprehendit Axiomata, ſeu communes animi notiones, quæ non ſolum in ſcientia propoſita ſed etiam in alijs omnibus vſqueadeo euidentes ſunt, vt ab eis nulla ratione diſſentire queat is, qui ipſa vocabula rectè perceperit, vt: Quæ eidem æqualia, & inter ſe ſunt æqualia.

Verum ſecundum nonnullos principium formale duplex exiſtit, nempe indemonſtrabile ſeu facile, vt definitio, poſtulatum, & axioma; demonſtrabile, vt Problema, Theorema, & omnis propoſitio. Principium vero materiale eſt punctum, linea, &c. Vid. & P. Chriſtophorus Clauius, nobiliſſimus Elementorum Euclidis Interpres.

Coloniæ Agrippinæ, anno 1607.
Iunij 8.

EVCLI-

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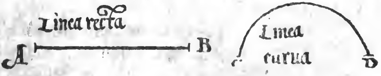
DEFINITIONES.

1.

PVnctum eſt, cuius pars Punctum nulla eſt.

2.

Linea verò, longitudo latitudinis expers.



3.

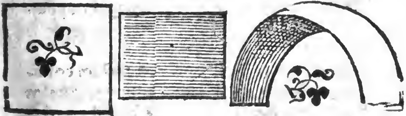
Lineæ autem termini, ſunt puncta.

4.

Recta linea eſt, quæ ex æquo ſua interiacet puncta.

5.

Superficies eſt, quæ longitudinem latitudinemque tantùm habet



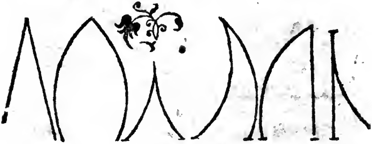
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6.

Superficiei extrema ſunt lineæ.

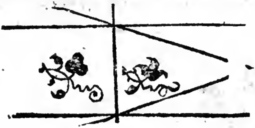
7.

Plana ſuperficies eſt, quæ ex æquo ſuas interiacet lineas.



8.

Planus angulus eſt duarum linearũ in plano ſe mutuò tangentium, & non indirectum iacēti



um alterius ad alteram inclinatio.

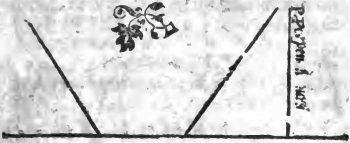
9.

Cùm autem quę angulum continent lineę, rectæ fuerint, rectilineus ille angulus appellatur.

10.

Cùm verò recta linea ſuper rectam conſiſtens lineam, eos, qui ſunt deinceps, angulos æquales inter ſe fecerit; rectus eſt vterque æqua-

æqualium angulorum : quæ inſiſtit recta linea, perpendicularis vocatur eius, cui inſiſtit.



11.

Obtuſus angulus eſt, qui recto maior eſt.

12.

Acutus verò, qui minor eſt recto.

13.

Terminus eſt, quod alicuius extremum eſt.



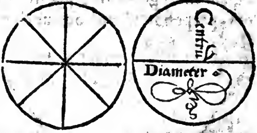
14.

Figura eſt, quæ ſub aliquo, vt aliquibus terminis comprehenditur.

15.

Circulus eſt, figura plana ſub vna linea cōprehenſa, quę peripheria appellatur; ad quā ab vno puncto eorum, quæ intra figuram

ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.



16.

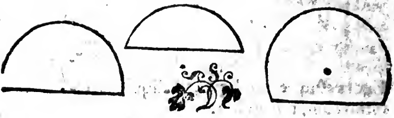
Hoc verò punctum, centrum circuli appellatur.

17.

Diameter autem circuli, eſt recta quædam linea per centrum ducta, & ex vtraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

18.

Semicirculus verò eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.



19.

Recti lineæ figuræ ſunt, quę ſub rectis lineis continentur.

20. Tri-



20. Trilateræ quidem, quæ ſub tribus.

21.

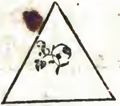
Quadrilateræ, quæ ſub quatuor.

22.

Multilateræ verò, quæ ſub pluribus, quàm quatuor, rectis lineis comprehenduntur.

23.

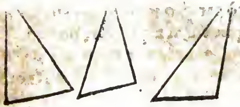
Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia.



24.

Iſoſceles autem eſt, quod duo tantùm æqualia habet latera.





25.

Scalenum verò eſt, quod tria inæ-

inæqualia habet latera.

26.

Ad hæc etiam, trilaterarum figurarum, re-ctangulum quidem triangulum eſt, quod rectum angulum habet.

27.

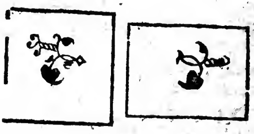
Amblygonium autem, quod obtuſum angulum habet.

28.

Oxygonium verò, quod tres habet acutos angulos.

29.

Quadrilaterorum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt.

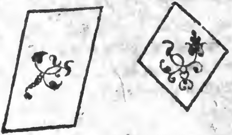


30.

Altera verò parte longior figura eſt, quę rectangula quidem, at æquilatera non eſt.

31. Rhom-

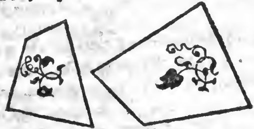
31.
Rhóbus autem, quę ęquilatera,



ſed rectangula non eſt.

32.
Rhomboides verò, quæ aduerſa & latera, & angulos habens inter ſe æquales, neque æquilatera eſt, neque rectangula.

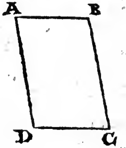
33.
Pręter has autē, reliquæ quadri



laterę figuræ, trapezia appellentur.

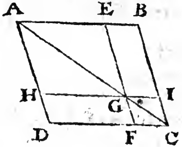
34.
Parallelę rectæ lineæ ſunt, quæ cùm in eodem ſint plano & ex vtraque parte in infinitum producantur, in neutram ſibi mutuò incidunt.

35.
Parallelo grammum eſt figura, quadrilatera, cuius bina appoſita latera ſunt parallela, ſeu æqui diſtantia.



36.

Cùm verò in parallelogrammo diameter ducta fuerit, duæq; lineæ lateribus parallelæ ſecantes diametrum in vno eodemque puncto, ita vt parallelogrammum ab hiſce parallelis in quatuor diſtribuatur parallelogramma; appellantur duo illa, per quæ diameter non tranſit, complementa; duo verò reliqua, per quæ diameter incedit, circa diametrum conſiſtere dicentur.



Petitiones ſiue Poſtulata.

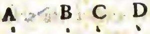
1.

Poſtuletur, vt à quouis puncto in quoduis punctum,



punctum, rectam lineam ducere cōcedatur.

2.



Et rectam lineam terminatam in continuū recta producere.

3.

Item quouis centro, & interuallo circulum deſcribere.



4.

Item quacunque magnitudine data, ſumi poſſe aliam magnitudinem vel maiorem, vel minorem.

Communes notiones ſiue axiomata.

1.

Quæ eidem æqualia, & inter ſe ſunt æqualia.

2.

Et, ſi æqualibus æqualia adiecta ſint, tota ſunt æqualia.

3.

Et, ſi æqualibus æqualia ablata ſint, quæ relinquuntur ſunt æqualia.

4.

Et, ſi inæqualibus æqualia adiecta ſint, tota ſunt inæqualia.

5.

Et, ſi ab inæqualibus æqualia ablata ſint, reliqua ſunt inæqualia.

6.

Et, quæ eiuſdem duplicia ſunt, inter ſe ſunt æqualia.

7.

Et quæ eiuſdem ſunt dimidia, inter ſe æqualia ſunt.

8.

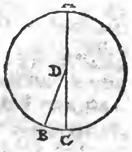
Et quæ ſibi mutuò congruunt, ea inter ſe ſunt æqualia.

9.

Et totum ſua parte maius eſt.

10.

Duæ lineæ rectæ non habent vnum, & idem ſegmentum commune.



11.

Duæ lineæ rectæ in vno puncto concurrentes,

rentes, ſi producantur ambæ, neceſſariò ſe mutuò in eo puncto interſecabunt.

12.

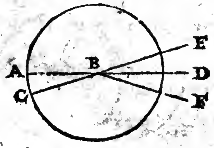
Item, omnes recti anguli ſunt inter ſe æquales.

13.

Et, ſi in duas rectas lineas altera recta incidens, internos, ad eaſdemque partes angulos, duobus rectis minores faciat; duæ illæ rectæ lineæ in infinitum productę ſibi mutuò incident ad eas partes, vbi ſunt anguli duobus rectis minores.

14.

Duæ rectæ lineæ ſpatium non comprehendunt.



15.

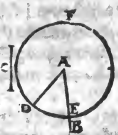
Si æqualibus inæqualia adiiciantur, erit totorum exceſſus, adiectorũ exceſſui æqualis

16.

Si inæqualibus æqualia adiungantur, erit totorum exceſſus, exceſſui eorum, quæ à principio erant, æqualis.

17. Si

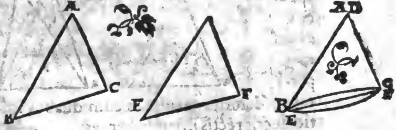
Problema 3. Pro-
ſitio 3.



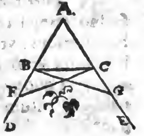
Duabus datis rectis lineis inæqualibus, de maiore æqualem minori rectam lineam detrahere.

Theorema 1. Propoſitio 4.

Si duo triangula duo latera duobus lateribus æqualia habeant, vtrunq; vtrique; habeant verò & angulum angulo ęqualem ſub æqualibus rectis lineis contentum: & baſin baſi æqualem habebunt; eritq; triangulum triangulo æquale, ac reliqui anguli reliquis angulis æquales erunt, vterque vtrique, ſub quibus æqualia latera ſubtenduntur.



Theorema 2. Pro-
poſitio 5.

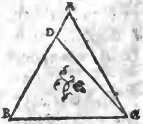


Iſoſceleum triangulorum, qui ad baſim ſunt anguli, inter ſe ſunt æquales; & ſi vlterius productæ ſint æquales illæ re-

ctæ

ctæ lineæ, qui ſub baſi ſunt anguli, inter ſe æquales erunt.

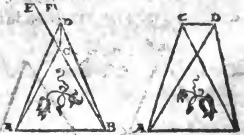
Theorema Problema 3. Propoſitio 6.



Si trianguli duo anguli æquales inter ſe fuerint; & ſub æqualibus angulis ſubtenſa latera æqualia inter ſe erunt.

Theorema 4. Propoſitio 7.

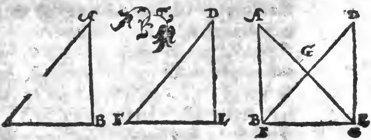
Super eadem recta linea, duabus eiſdem rectis lineis aliæ duæ rectæ lineæ æquales, vtraque vtrique, non conſtituentur, ad aliud atq; aliud punctū, ad eaſdem partes, eoſdemque terminos cum duabus initio ductis rectis lineis habentes.



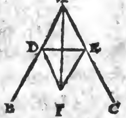
Theorema 5. Propoſitio. 8.

Si duo triangula duo latera habuerint duobus lateribus, vtrunque vtrique, æqualia: habuerint verò & baſim baſi æqualem; angulum quoque ſub æqualibus rectis lineis contentum angulo æqualem habebunt.

Pro-

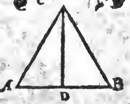


Problema 4. Propoſitio 9.



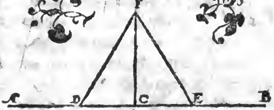
Datum angulum rectilineum biſariam ſecare.

Problema 5. Propoſitio 10.



Datam rectam lineam finitam biſariam ſecare.

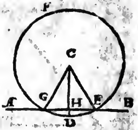
Problema 6. Propoſitio 11.



Data recta linea, à puncto in ea dato, rectam lineam ad angulos rectos excitare.

Pro-

Problema 7. Propoſitio 12.



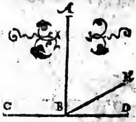
Super datam rectam lineam infinitam, à dato puncto, quod in ea non eſt, perpendicularem rectam deducere.

Theorema 6. Propoſitio 13.



Cùm recta linea ſuper rectam cõſiſtens lineam angulos facit, aut duos rectos, aut duobus rectis æquales efficiet.

Theorema 7. Propoſitio 14.



Si ad aliquam rectam lineam, atq; ad eius punctum, duæ rectæ lineæ non ad eaſdem partes ductæ, eos, qui ſunt deinceps, angulos duobus rectis æquales fecerint, indirectũ erunt inter ſe ipſe rectæ lineæ.

Theorema 8. Propoſitio 15.

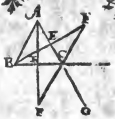


Si duæ rectæ lineæ ſe mutuò ſecuerint, angulos, qui ad verticem ſunt, æquales inter ſe efficient.

Theo-

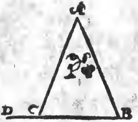
Theorema 9. Propoſitio 16.

Cuiuſcunque trianguli vno latere producto, externus angulus vtroque interno & oppoſito maior eſt.



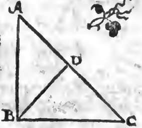
Theorema 10. Propoſitio 17.

Cuiuſcunque trianguli duo anguli duobus rectis ſunt minores, omnifariam ſumpti.



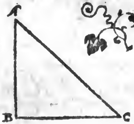
Theorema 11. Propoſitio.

Omnis trianguli maius latus maiorem angulũ ſubtendit.

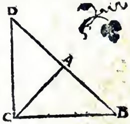


Theorema 12. Propoſitio 19.

Omnis trianguli maior angulus maiori lateri ſubtenditur.

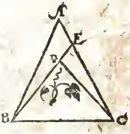


Theorema 13. Propoſitio 20.



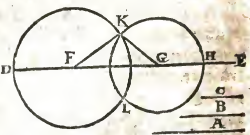
Omnis trianguli duo latera, reliquo ſunt maiora, quomodocunque aſſumpta.

Theorema 14. Propoſitio 21.



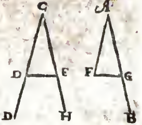
Si ſuper trianguli vno latere, ab extremitatibus duæ rectæ lineæ, interius conſtitutæ fuerint; hæ conſtitutæ reliquis trianguli duobus lateribus minores quidem erunt; minorem verò angulū continebunt.

Problema 8. Propoſitio 22.



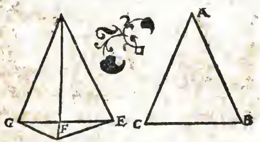
Ex tribus rectis lineis, quæ ſunt trib. datis rectis lineis æquales, triangulum conſtituere. Oportet autem duas reliqua eſſe maiores omnifariam ſumptas: quoniam vnuſcuiuſque trianguli duo latera omnifariam ſumpta, reliquo ſunt maiora.

Problema 9. Propoſitio 23.



Ad datam rectam lineã, datumq; in ea punctum, dato angulo rectilineo æqualem angulum rectilineum conſtituere.

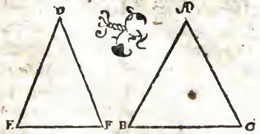
Theorema 15. Propoſitio 24.



Si duo triangula duo latera duobus lateribus ęqualia habuerint, vtrũq; vtrique, angulum verò angulo maiorẽ ſub æqualib. rectiſlineis cõtentum: & baſin baſi maiorẽ habebunt.

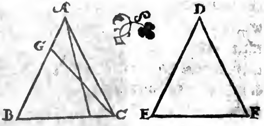
Theorema 16. Propoſitio. 25.

Si duo triangula duo latera duobus lateribus æqualia habuerint, vtrunque vtrique: baſin verò baſi maiorẽ: & angulum ſub æqualibus rectiſlineis contentum angulo maiorem habebunt.



Theorema 17. Propoſitio 26.

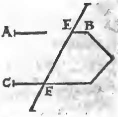
Si duo triangula duos angulos duobus angulis æquales habuerint, vtrunque vtrique; vnumque latus vni lateri æquale, ſiue quod æqualibus adiacet angulis, ſeu quod vni æqualium angulorum ſubtenditur: & reliqua latera reliquis lateribus æqualia vtrunq; vtriq;



& reliquum angulum reliquo angulo æqualem habebunt.

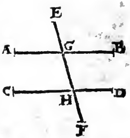
Theorema 18. Propoſitio 27.

Si in duas rectas lineas recta incidens linea alternatim angulos æquales inter ſe fecerit: parallelæ erunt inter ſe illæ rectæ lineæ



Theorema 19. Propoſitio 28.

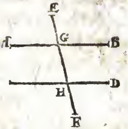
Si in duas rectas lineas recta incidens linea, externum angulum interno, & oppoſito, & ad eaſdem partes, æqualem fe-



cerit,

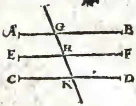
cerit, aut internos, & ad eaſdem partes duobus rectis æquales: parallelæ erunt inter ſe ipſæ rectæ lineæ.

Theorema 20. Propoſitio 29.



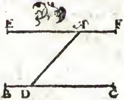
In parallelas rectas lineas recta incidens linea: & alternatim angulos inter ſe æquales efficit, & externum interno & oppoſito, & ad eaſdem partes æqualem, & internos & ad eaſdem partes duobus rectis æquales facit.

Theorema 21. Propoſitio 30.



Quæ eidem rectæ lineæ, parallelæ, & inter ſe ſunt parallelæ.

Problema 10. Propoſitio 31.

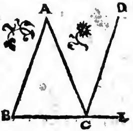


A dato puncto datæ rectæ lineæ parallelam rectam lineam ducere.

Theorema 22. Propoſitio 31.

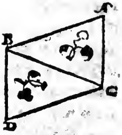
Cuiuſcunque trianguli vno latere alterius

producto: externus angelus duobus internis & oppoſitis eſt æqualis. Et trianguli tres interni anguli duobus ſunt rectis æquales.



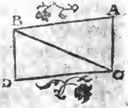
Theorema 23 Propoſitio 33.

Rectæ lineæ quæ æquales & parallelas lineas ad partes eaſdem coniungunt, & ipſæ æquales & parallelæ ſunt.



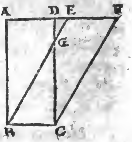
Theorema 24. Propoſitio 34.

Parallelogrammorum ſpatiorum æqualia ſunt inter ſe; quæ ex aduerſo, & latera, & anguli: atque illa biſariam ſecat diameter.



Theorema 25. Propoſitio 35.

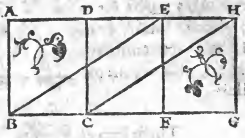
Parallelogramma ſuper eadem baſi, & in eiſdem parallelis conſtituta inter ſe ſunt æqualia.



Theo-

Theorema 26. Propoſitio 36.

Parallelogramma ſuper æqualibus baſibus, in eiſdẽ parallelis cõſtituta inter ſe ſunt æqualia.



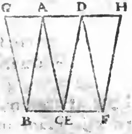
Theorema 27. Propoſitio 37.

Triangula ſuper eadem baſi conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.



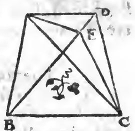
Theorema 28. Propoſitio 38.

Triangula ſuper æqualibus baſibus conſtituta, & in eiſdem parallelis, inter ſe ſunt æqualia.

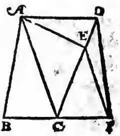


Theorema 29. Propoſitio 39.

Triangula ęqualia ſuper eadem baſi, & ad eaſdem partes conſtituta: & in eiſdem ſunt parallelis.

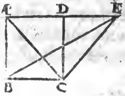


Theorema 30, Propoſitio 40.



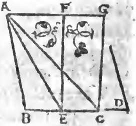
Triangula æqualia ſuper æqualibus baſibus, & ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis.

Theorema 31. Propoſitio 41.



Si parallelogrammum cum triangulo eandem baſin habuerit, in eiſdemque fuerit parallelis, duplum erit parallelogrammum ipſius trianguli.

Problema 11. Proſitio 42.



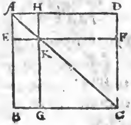
Dato triangulo æquale parallelogrammum cõſtituere in dato angulo rectilineo.

Theorema 32. Propoſitio 34.

In omni parallelo grammo, complementa eorum,

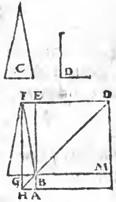
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eorum, quæ circa diametrum ſunt, parallelogrãmorum, inter ſe ſunt æqualia.



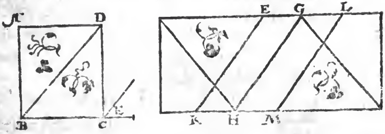
Problema 12. Propoſitio 44.

Ad datam rectam lineam, dato, triangulo, æquale parallelogrammum applicare, in dato angulo rectilineo.



Problema 13. Propoſitio 45.

Dato rectilineo, æquale parallelogrammũ conſtituere in dato angulo rectilineo.



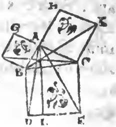
Problema 14. Propoſitio 46.

A data recta linea quadratum deſcribere.



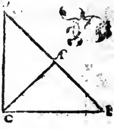
Theorema 33. Propoſitio 47.

In rectangulis triangulis, quadratum, quod à latere rectum angulum ſubtendente deſcribitur, æquale eſt eis, quæ à lateribus rectum angulum continentibus deſcribuntur, quadratis.



Theorema 34. Propoſitio 48.

Si quadratum, quod ab vno laterum trianguli deſcribitur, æquale ſit eis, qui à reliquis trianguli lateribus deſcribuntur, quadratis: angulus comprehenſus ſub reliquis duobus trianguli lateribus, rectus eſt.



FINIS ELEMENTI I.

EVCLI.

EVCLIDIS ELEMENTVM SECVNDVM.

DEFINITIONES.

1.



Omne parallelogrammum rectangulum cõtineri dicitur ſub rectis duabus lineis, quæ rectum comprehendunt angulum.

2.

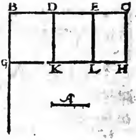


In omni parallelogrammo ſpatio, vnum quodlibet eorum, quæ circa diametrum illius ſunt, parallelogrammorum, cum duobus complementis, Gnomon vocetur.

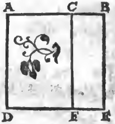
Theorema 1. Propoſitio 1.

Si fuerint duæ rectæ lineæ, ſeceturq; ipſarũ altera in quotcunque ſegmenta rectangulũ com-

comprehenſum ſub illis duabus rectis lineis, æquale eſt eis quæ ſub inſecta & quolibet ſegmétorum comprehenduntur, rectangulis.



Theorema 2. Propoſitio 2.



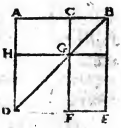
Si recta linea ſecta ſit vtcunque: rectangula, quæ ſub tota, & quolibet ſegmentorum compræhenduntur, æqualia ſunt ei, quod à toto fit, quadrato.

Theorema 3. Propoſitio. 3.

Si recta linea ſecta ſit vtcunque rectangulum ſub tota, & vno ſegmentorum comprehenſum, æquale eſt illi, quod ſub ſegmentis comprehenditur rectangulo, & illi, quod à prædicto ſegméto deſcribitur, quadrato.



Theorema 4. Propoſitio 4.

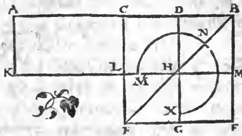


Si recta linea ſecta ſit vtcunque quadratum, quod à tota deſcribitur, æquale eſt & illis, quæ à ſegmentis

tis deſcribuntur, quadratis; & ei quod bis ſub ſegmentis comprehenditur, rectangulo.

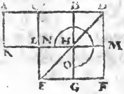
Theorema 5. Propoſitio 5.

Si recta linea ſecetur in æqualia, & non æqualia: rectangulum ſub inæqualibus ſegmentis totius comprehenſum, vnà cũ quadrato, quod ab intermedia ſectionum, æquale eſt ei, quod à dimidia deſcribitur, quadrato.



Theorema 6. Propoſitio 6.

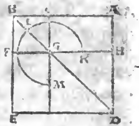
Si recta linea bifariam ſecetur, & illi recta quædam linea in rectum adijciatur: rectangulum comprehenſum ſub tota cum adiecta, & adiecta, vnà cum quadrato à dimidia, æquale eſt quadrato à linea, quæ tum ex dimidia, tum ex adiecta componitur, tanquam ab vna, deſcripto.



Theorema 7. Propoſitio 7.

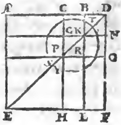
Si recta linea ſecetur vtcunque; quod à tota, quod-

quodque ab vno ſegmentorum, vtraq; ſimul quadrata, æqualia ſunt & illi, quod bis ſub tota, & dicto ſegmento comprehenditur, rectãgulo; & illi quod à reliquo ſegmento fit, quadrato.



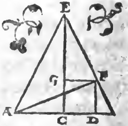
Theorema 8. Propoſitio 8.

Si recta linea ſecetur vtcunque; rectangulũ quater comprehenſum ſub tota, & vno ſegmentorum, cum eo, quod à reliquo ſegmẽto fit, quadrato, ęquale eſt ei, quod à tota, & dicto ſegmẽto, tanquam ab vna linea deſcribitur, quadrato.



Theorema 9. Propoſitio 9.

Si recta linea ſecetur in æqualia & non æqualia: quadrata, quæ ab inæqualibus totius ſegmentis fiunt, ſimul duplicia ſunt, & eius, quod à dimidia, & eius, quod ab intermedia ſectionum fit, quadratorum.

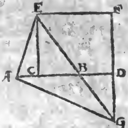


Theorema 10. Propoſitio 10.

Si recta linea ſecetur bifariã, adijciatur aũt ei in

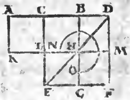
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ei in rectũ quæpiam recta linea: quod à tota cum ad iuncta, & quod ab adiuncta, vtraque ſimul quadrata; duplicia ſunt, & eius, quod à dimidia, & eius, quod à compoſita ex dimidia & adiuncta, tanquam ab vna, deſcriptum ſit, quadratorum.



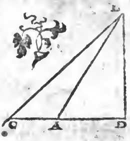
Problema 1. Propoſitio 11.

Datam rectam lineã ſecare, vt comprahenſum ſub tota, & altero ſegmẽtorum rectangulum, æquale ſit ei, quod à reliquo ſegmento fit, quadrato.



Theorema 11. Propoſitio 12.

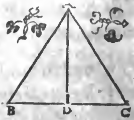
In amblygonijs triãgulis, quadratum, quod fit à latere angulum obtuſum ſubtendente, maius eſt quadratis, quæ fiunt à lateribus obtuſum angulum comprehendentibus; rectangulo bis comprahenſo, & ab vno laterum, quæ ſunt circa obtuſum angulum, in quod cũ protractum fuerit, cadit perpendicularis, & ab aſſumpta exterius linea ſub perpendiculari prope angulum obtuſum.



Theo-

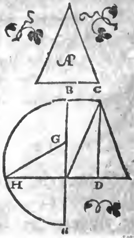
Theorema 12. Propoſitio 28.

In oxygonijs triangulis, quadratum à latere angulum acutum ſubtendente, minus eſt quadratis, quæ fiunt à lateribus acutum angulum comprehendentibus; rectangulo bis comprehenſo; & vno laterum, quæ ſunt circa acutum angulum, in quod perpẽdicularis cadit, & ab aſſumpta interius linea ſub perpendiculari prope acutum angulum.



Problema 2. Propoſitio 14.

Dato rectilineo æquale quadratum conſtituere.



FINIS ELEMENTI II.

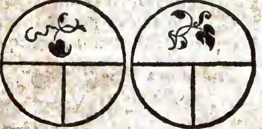
EVCLI-

EVCLIDIS ELEMENTVM TERTIVM.

DEFINITIONES.

1.

Aequales circuli ſunt, quorum diametri ſunt æquales; vel quorum, quæ ex centris, rectæ lineæ ſunt æquales.



2.

Recta linea circulum tangere dicitur, quæ cùm circulum tangat; ſi producatur, circulum non ſecat.



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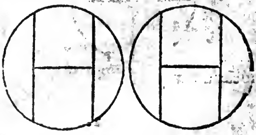
3.

Circuli ſeſe mutuò tangere dicuntur: qui ſeſe mutuò tangentes, ſeſe mutuò non ſecant.



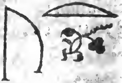
4.

In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cùm perpendiculares, quæ à centro in ipſis ducuntur, ſunt æquales. Lõgius autem abeſſe illa dicitur, in quã maior perpendicularis cadit.



5.

Segmentum circuli eſt figura, quæ ſub recta linea, & circuli peripheria comprehenditur.



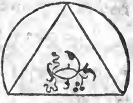
6.

Segmenti autem angulus eſt, qui ſub recta linea

linea, & circuli peripheria comprehenditur.

7.

In ſegmento autem angulus eſt, cùm in ſegmenti peripheria ſumptum fuerit quodpiam punctum, & ab illo in terminos rectæ eius lineæ quæ ſegmenti baſis eſt, adiunctæ fuerint, rectæ lineæ: is, inquam, angulus ab adiunctis illis lineis comprehenſus.



8.

Cùm verò comprehendentes angulum rectæ lineę aliquam aſſumant peripheriam, illi angulus inſiſtere dicitur.



9.

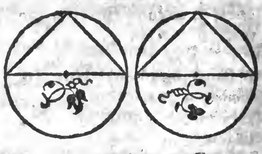
Sector autem cirouli eſt, cùm ad ipſius circuli cẽtrum conſtitutus fuerit angulus, compræhenſa nimirum figura, & à rectis lineis angulum continentibus, & à peripheria ab illis aſſumpta.



10.

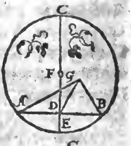
Similia circuli ſegmenta ſunt, quæ angulos

capiunt æquales aut in quibus anguli inter ſe ſunt æquales.



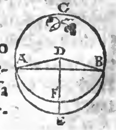
Problema 1. Propoſitio 1.

Dati circuli centrum reperire.



Theorema 1. Propoſitio 2.

Si in circuli peripheria duo quælibet puncta accepta fuerint; recta linea, quæ ad ipſa puncta adiungitur, intra circulum cader.

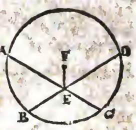


Theorema 2. Propoſitio. 3.

Si in circulo recta quædam linea per centrum extenſa quandam non per centrum extenſam bifariam ſecet: & ad angulos rectos ipſam ſecabit: Et ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.

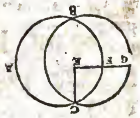


Theorema 3. Propoſitio 4.



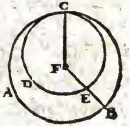
Si in circulo duæ rectæ lineæ ſeſe mutuò ſecent, nō per centrum extenſæ; ſeſe mutuò bifariam non ſecabunt.

Theorema 4. Propoſitio 5.



Si duo circuli ſeſe mutuò ſecent; non erit illorum idem centrum.

Theorema 5. Propoſitio 6.



Si duo circuli ſeſe mutuò interius tangent, eorum non erit idem centrum.

Theorema 6. Propoſitio 7.

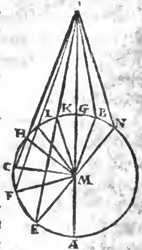
Si in diametro circuli quodpiam ſumatur punctum, quod circuli centrum non ſit, ab eoq; puncto in circulum quædam rectæ lineæ cadant; maxima quidem erit ea in qua centrū; minima verò reliqua; aliarum verò propinquior illi, quæ per centrum du-



citur, remotiore ſemper maior eſt. Duæ autem ſolùm rectæ lineæ æquales ab eodem puncto in circulum cadunt, ad vtraſq; partes minimæ, vel maximæ.

Theorema 7. Propoſitio 8.

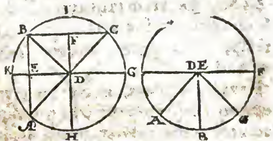
Si extra circulum ſumatur punctum quodpiam, ab eoque puncto ad circulum deducantur rectæ quædam lineæ, quarum vna quidem per centrum protendatur, reliquæ verò vt libet: in cauam peripheriam cadentium rectarum linearum minima quidem eſt illa, quæ per centrum ducitur: aliarum autem propinquior ei, quæ per centrum tranſit, remotiore ſemper maior eſt: in conuexam verò peripheriã cadentium rectarum linearũ minima quidem eſt illa, quæ inter punctum, & diametrum interponitur: aliarum autem, ea, quæ propinquior eſt minimæ, remotiore ſemper minor eſt. Duæ autem tantùm rectæ lineæ æquales ab eo puncto in ipſum circulum cadunt, ad vtraſque partes minimæ, vel maximæ.



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Theorema 8. Propoſitio 9.

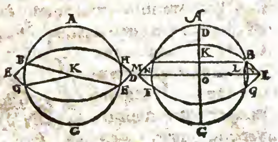
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unctum centrum eſt ipſius circuli.

Theorema 9. Propoſitio 10.

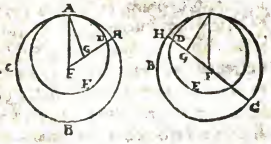
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Theorema 10. Propoſitio 11.

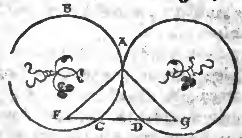
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ſuerint eorum centra; ad eorum centra adiuncta recta linea, & producta, in cõtactum circulorum cadet.

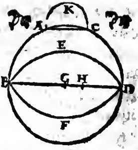
Theorema 11. Propoſitio 12.

Si duo circuli ſeſe exterius contingant, linea recta, quæ ad centra eorum adiungitur, per contactum illum traſibit.



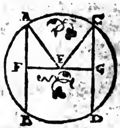
Theorema 12. Propoſitio 13.

Circulum circulum non tangit in pluribus punctis, quam vno, ſiue intus ſiue extra tangat.



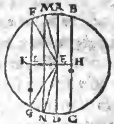
Theorema 13. Propoſitio 14.

In circulo æquales rectæ lineæ æqualiter diſtant à centro. Et quæ æqualiter diſtant à centro, æquales ſunt inter ſe.



Theo-

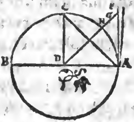
Theorema 14. Propoſitio 15.



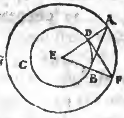
In circulo maxima quidem linea eſt diameter: aliarum autem propinquior centro, remotiore ſemper maior.

Theorema 15. Propoſitio 16.

Quæ ab extremitate diametri cuiuſque circuli ad angulos rectos ducitur, extra ipſum circulum cadet; & in locum inter ipſam rectam lineam, & peripheriam comprehenſum, altera recta linea non cadet. Et ſemicirculi quidem angulus quouis angulo acuto rectilineo maior eſt, reliquus autẽ minor.



Problena 12. Propoſtio 17.



A dato puncto rectam lineam ducere, quę datum tangat circulum.

Theorema 16. Propoſitio 18.

Si circulum tangat recta quæpiam linea, à

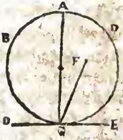
 centro

centro autem ad contactum adiungatur recta quædam linea: quæ adiuncta fuerit, ad ipſam contingentem, perpendicularis erit.



Theorema 17. Propoſitio 19.

Si circulum tetigerit recta quæpiam linea, à contactu autem recta linea ad angulos rectos ipſi tãgenti excitetur; inexcitata erit centrum circuli.



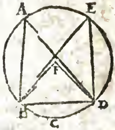
Theorema 18. Propoſitio 20.

In circulo, angulus ad centrum duplex eſt anguli ad peripheriam, cũ fuerit eadem peripheria baſis angulorum.



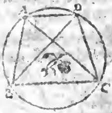
Theorema 19. Propoſitio 21.

In circulo, qui in eodem ſegmẽto ſunt, anguli, ſunt inter ſe æquales.



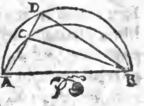
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Theorema 20. Propoſitio 22.

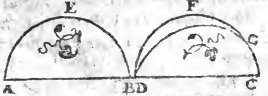


Quadrilaterorum in circulis deſcriptorum anguli, qui ex aduerſo, duob⁹ rectis ſunt æquales.

Theorema 21. Propoſitio 23.



Super eadem recta linea duo ſegmenta circulorũ ſimilia, & inæqualia non conſtituentur ad eaſdem partes.



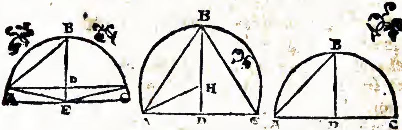
Super æqualibus rectis lineis, ſimilia circulorum ſegmenta, ſunt inter ſe æqualia.

Problema 3. Propoſitio 25.

Circuli ſegmento dato, deſcribere circulũ, cuius

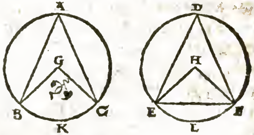
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cuius eſt ſegmentum.



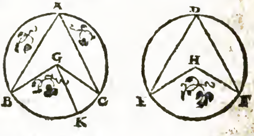
Theorema 23. Propoſitio 26.

In æqualibus circulis, æquales anguli æqualibus peripheriis inſiſtunt, ſiue ad cẽtra, ſiue ad peripherias conſtituti, inſiſtant.



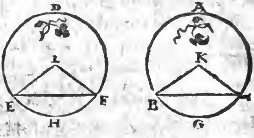
Theorema 24. Propoſitio 27.

In æqualibus circulis, anguli, qui æqualibus peripherijs inſiſtunt, ſunt inter ſe æquales, ſiue ad centra, ſiue ad peripherias conſtituti, inſiſtant.



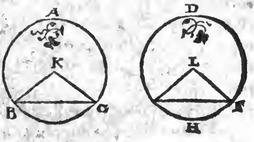
Theorema 25. Propoſitio 28.

In æqualibus circulis, æquales rectæ lineæ, æquales peripherias auferunt; maiorē quidē maiori, minorem autem minori.



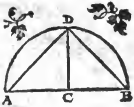
Theorema 26. Propoſitio 29.

In æqualibus circulis æquales peripherias, æquales rectæ lineæ ſubtendunt.



Problema 4. Propoſitio 30.

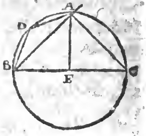
Datam peripheriam bifariam ſecare.



Theorema 27. Propoſitio 31.

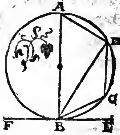
In circulo angulus, qui in ſemicirculo, rectus

ctus eſt: qui autem in maiore ſegmento, minor recto: qui verò in minore ſegmento, maior eſt recto. Et inſuper angulus maioris ſegmenti, recto quidem maior eſt, minoris autem ſegmenti angulus, minor eſt recto.



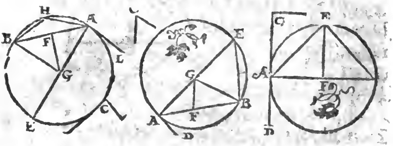
Theorema 28. Propoſitio 32.

Si circulum tetigerit aliqua recta linea, à contactu autẽ producatur quædam recta linea circulum ſecans: anguli, quos ad contingentem facit, ęquales ſunt ijs, qui in alternis circuli ſegmẽtis conſiſtunt, angulis.



Problema 5. Propoſitio 33.

Super data recta linea deſcribere ſegmentũ circuli, quod capiat angulum æqualem dato angulo rectilineo.



Pro-

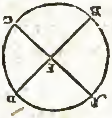
Problema 6. Propoſitio 34.

A dato circulo ſegmentum abſcindere, capiens angulum æqualem dato angulo rectilineo.

Theorema 29. Propoſitio 35.

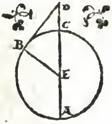
Si in circulo duæ rectæ lineæ ſeſe mutuò ſecuerint; rectangulum comprehenſum ſub ſegmentis vni°, æquale eſt ei, quod ſub ſegmentis alterius comprehenditur, rectangulo.

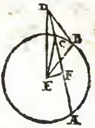




Theorema 30. Propoſitio 36.

Si extra circulū ſumatur pūctū aliquod, ab eoque in circulū cadant duæ rectæ lineæ; quarum altera quidem circulū ſecet, altera verò

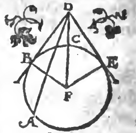




verò tangat: quod ſub tota ſecante, & exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur, rectangulum; æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulum ſumatur punctum aliquod ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat; ſit autem, quod ſub tota ſecante, & exterius inter punctum, & conuexam peripheriam aſſumpta, comprehenditur, rectangulum, æquale ei, quod ab incidente deſcribitur, quadrato; incidens ipſa circulum tanget.



FINIS ELEMENTI III.

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EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

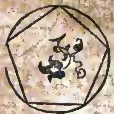
1.

Figura rectilinea in figura rectilinea inſcribi dicitur, cùm ſinguli eius figuræ; quæ inſcribitur, anguli, ſingula latera eius, in qua inſcribitur, tangunt.



2.

Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius, quæ circumſcribitur, latera ſingulos eius figuræ angulos tetigerint, circum quam illa deſcribitur.





3.

Figura rectilinea in circulo inſcribi dicitur, quum ſinguli eius figuræ, quæ inſcribitur,

anguli tetigerint circuli peripheriam.

4.

Figura verò rectilinea circa circulum deſcribi dicitur, quùm ſingula latera eius, quæ circum ſcribitur, circuli peripheriam tangunt.

5.

Similiter & circulus in figura rectilinea inſcribi dicitur, quũ circuli peripheria ſingula latera tangit eius figuræ, cui inſcribitur.

6.

Circulus autem circum figuræ deſcribi dicitur, quum circuli peripheria ſingulos tangit eius figuræ, quam circumſcribit, angulos.

7.



Recta linea in circulo accommodari, ſeu coaptari dicitur, quum eius extrema in circuli peripheria fuerint.

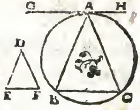
Problema 1. Propoſitio 1.



In dato circulo, rectam lineam accommodare æqualem datæ rectæ lineę, quæ circuli diametro nõ ſit maior.

Problema 2. Propoſitio 2.

In dato circulo, triangulum deſcribere, dato triangulo æquiangulum.



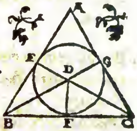
Problema 3. Propoſitio 3.

Circa datum circulum triangulum deſcribere, dato triangulo æquiangulum.



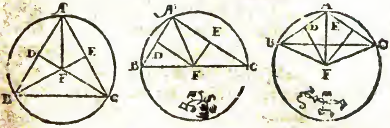
Problema 4 Propoſitio 4.

In dato triangulo circulum inſcribere.



Problema 5. Propoſitio 5.

Circa datum triangulū, circulū deſcribere.



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Problema 6. Propoſitio 6.

In dato circulo quadratum deſcribere.



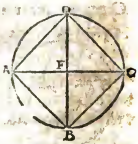
Problema 7. Propoſitio. 7.

Circa datum circulum, quadratum deſcribere.



Problema 8. Propoſitio 8.

In dato quadrato circulum inſcribere.



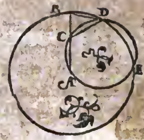
Problema 9. Propoſitio 9.

Circa datum quadratũ, circulum deſcribere.

Pro-

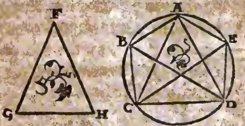
Problema 10. Propoſitio. 10.

Iſoſceles triangulum cõſtituere; quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.



Problema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterum, & æquiangulum inſcribere.



Problema 12. Propoſitio 12.

Circa datum circulum, pentagonum, æquilaterum & æquiangulum deſcribere.



Problema 13. Propoſitio 13.

In dato pentagono æquilatero, & æquiangulo circulum inſcribere.



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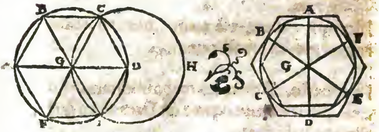
Problema 14 Propoſitio 14.



Circa datum pentagonũ æquilaterum & æquiangulum, circulum deſcribere.

Problema 15. Propoſitio 15.

In dato circulo hexagonum & æquilaterũ, & æquiangulum inſcribere.



Problema 16. Propoſitio 16.

In dato circulo quinti decagonum & æquilaterũ, & æquiangulum deſcribere.



FINIS ELEMENTI IV.

Google

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

PArs eſt magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem eſt maior minoris, cùm minor metitur maiorem.

3.

Proportio, eſt duarum magnitudinum eiuſdem generis mutua quædam ſecundum quãtitalem habitudo.

4.

Proportionalitas verò eſt proportionũ ſimilitudo.

5.

Proportionem habere inter ſe magnitudines dicuntur, quæ poſſunt multiplicatæ ſeſe mutuò ſuperare.

6.

In eadẽ proportione magnitudines dicũtur eſſe, prima ad ſecundam, & tertia ad quartam, cùm primæ & tertiæ æquè multiplicia, à ſecundæ & quartæ æquè multiplicibus,

qualiſcunque ſit hæc multiplicatio, vtrunque ab vtroque; vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt; ſi ea ſumantur, quæ inter ſe reſpondent.

7.

Eandem autem proportionem habentes magnitudines, proportionales vocentur.

8.

Cùm verò æquè multiplicium, multiplex primæ magnitudinis exceſſerit multiplicē ſecundæ; at multiplex tertiæ non exceſſerit multiplicem quartæ; tunc prima ad ſecundam, maiorem proportionem habere dicetur, quàm tertia ad quartam.

9.

Proportinoalitas autem in tribus terminis pauciſsimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales, fuerint, prima ad tertiam, duplicatam proportionem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam proportionem habere dicitur eius, quam habet ad ſecundam: & ſemper deinceps, vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles proportione magnitudines dicuntur, antecedentes quidem ante-

antecedentibus, conſequentes verò conſequentibus.

12.

Alterna proportio, eſt ſumptio antecedentis ad antecedentem,&conſequentis ad conſequenrem.

13.

Inuerſa proportio, eſt ſumptio conſequentis, ceu antecedentis, ad antecedentem, velut ad ipſam conſequentem.

14.

Compoſitio proportionis, eſt ſumptio antecedentis cum conſequente, ceu vnius ad ipſam conſequentem.

15.

Diuiſio proportionis, eſt ſumptio exceſſus, quo conſequentem ſuperat antecedens, ad ipſam conſequentem.

16.

Conuerſio proportionis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſam conſequentem.

17.

Ex æqualitate proportio eſt, ſi plures duabus ſint magnitudines,& his aliæ multitudine pares, quæ binæ ſumantur, & in eadem proportione: quum vt in primis magnitudinibus prima ad vltimam, ſic & in ſecundis magnitudinibus prima ad vltimam ſeſe habuerit.

Vel aliter, ſumptio extremorum per ſubductionem mediorum.

18.

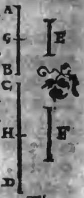
Ordinata proportio eſt, cùm fuerit, quemadmodum antecedens ad conſequentem, ita antecedens ad conſeqnentem: fuerit etiam vt conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

19.

Perturbata autem proportio eſt, cùm tribꝰ poſitis magnitudinibus, & alijs quæ ſint his multitudine pares, vt in primis quidem magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: vt autem aliud quidpiam ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

Theorema 1. Propoſitio 1.

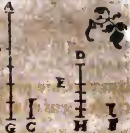
Si ſint quotcunq; magnitudines, quotcunque magnitudinum æqualium numero, ſingulæ ſingularum, æquè multiplices; quàm multiplex eſt vnius vna magnitudo, tam multipleces erunt & omnes omnium.



Theo-

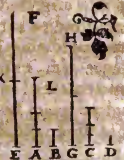
Theorema 2. Propopoſitio 2.

Si prima ſecundæ æquè fuerit multiplex, atque tertia quartæ; fuerit autem & quinta ſecundæ æquè multiplex, atque ſexta quartæ: erit & compoſita prima cũ quinta, ſecundæ æquè multiplex; atq; tertia cum ſexta, quartæ.



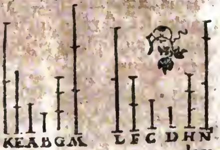
Theorema 3. Propoſitio 3.

Si ſit prima ſecundæ æquè multiplex, atque tertia quartæ; ſumantur autem æquè multiplices primæ, & tertiæ; erit & ex æquo, ſumptarum vtraque vtriuſque æquè multiplex; altera quidem ſecundæ, altera autem quartæ.

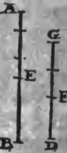


Theorema 4. Propoſitio 4.

Si prima ad ſecundã, eandem habuerit proportionẽ, & tertia ad quartam: etiam æquè multiplices primę & tertię, ad ęquè multiplices ſecundæ & quartæ, iuxta quãmuis multiplicationẽ, eandem

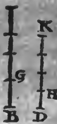


dem habebunt propoſitionem; ſi, prout inter ſe reſpondent, ita ſumptæ fuerint.



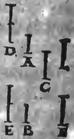
Theorema 5. Propoſitio 5.

Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.



Theorema 6. Propoſitio 6.

Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractæ quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut æquales ſunt, aut æquè ipſarum multiplices.



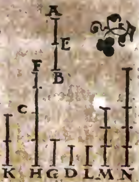
Theorema 7. Propoſitio 7.

Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propoſitio 8

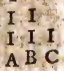
Inæqualium magnitudinum maior ad eandem

dem, maiorem proportionem habet: quàm minor: & eadem ad minorem, maiorem proportionem habet, quàm ad maiorem.



Theorema 9. Propoſitio 9.

Quæ ad eandem, eandem habent proportionem, æquales ſunt inter ſe: & ad quas eadem, eandem habet proportionem, eæ quoq; ſunt inter ſe æquales.



Theorema 10. Propoſitio 10.

Ad eandem magnitudinem proportionem habentium, quę maiorem proportionem habet, illa maior eſt, ad quam autem eadem maiorem proportionem habet, illa minor eſt.



Theorema 11. Propoſitio 11.

Quæ eidem ſunt eædem proportiones, & inter ſe ſunt eædem.

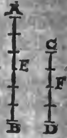


Theorema 18. Propoſitio 18.



Si diuiſæ magnitudines ſint proportionales, hæ quoque compoſitæ proportionales erunt.

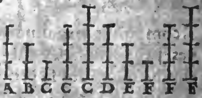
Theorema 19. Propoſitio 19.



Si quemadmodum totum ad totum, ita ablatum ſe habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum, ſe habebit.

Theorema 20. Propoſitio 20.

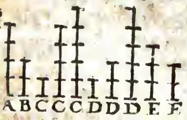
Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur; ex æquo autem prima quàm tertia maior fuerit; erit & quarta, quam ſexta, maior. Quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, quoque minor erit.



Theo-

Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur, fueritque perturbata earum proportio: ex æquo autem prima, quàm tertia, maior fuerit; erit & quarta, quàm ſexta, maior: quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ; ſin illa minor, hæc quoque minor erit.



Theorema 22. Propoſitio 22.

Si ſint quodcunque magnitudines, & aliæ ipſis æquales numero, quæ binæ in eadē proportione ſumantur: & ex æqualitate in eadem proportione erunt.



Theorema 23. Propoſitio 23.

Si ſint tres magnitudines, aliæque ipſis æqua-

Problema 6. Propoſitio 6.

In dato circulo quadratum deſcribere.



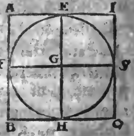
Problema 7. Propoſitio. 7.

Circa datum circulum, quadratum deſcribere.



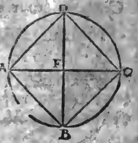
Problema 8. Propoſitio 8.

In dato quadrato circulum inſcribere.



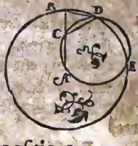
Problema 9. Propoſitio 9.

Circa datum quadratũ, circulum deſcribere.



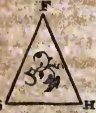
Problema 10. Propoſitio. 10.

Iſoſceles triangulum cõſtituere; quod habeat vtrunque eorum, qui ad baſin ſunt, angulorum, duplum reliqui.



Problema 11. Propoſitio 11.

In dato circulo, pentagonũ æquilaterum, & æquiangulum inſcribere.





Problema 12. Propoſitio 12.

Circa datum circulum, pentagonum, æquilaterum & æquiangulum deſcribere.

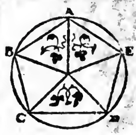


Problema 13. Propoſitio 13.

In dato pentagono æquilatero, & æquiangulo circulum inſcribere.



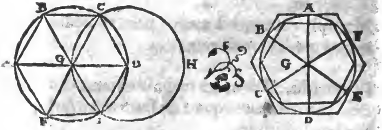
Problema 14 Propo-ſitio 14.



Circa datum pentagonū æquilaterum & æquiangulum, circulum deſcribere.

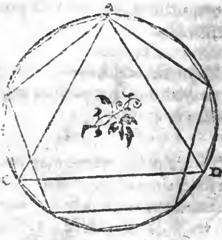
Problema 15. Propoſitio 15.

In dato circulo hexagonum & æquilaterū, & æquiangulum inſcribere.



Problema 16. Propoſitio 16.

In dato circulo quinti decagonum & æquilaterū, & æquiangulum deſcribere.



FINIS ELEMENTI IV.

EVCLIDIS ELEMENTVM QVINTVM.

DEFINITIONES.

1.

PArs eſt magnitudo magnitudinis minor maioris, quum minor metitur maiorem.

2.

Multiplex autem eſt maior minoris, cùm minor metitur maiorem.

3.

Proportio, eſt duarum magnitudinum eiuſdem generis mutua quædam ſecundum quãti talem habitudo.

4.

Proportionalitas verò eſt proportionũ ſimilitudo.

5.

Proportionem habere inter ſe magnitudines dicuntur, quæ poſſunt multiplicatæ ſeſe mutuò ſuperare.

6.

In eadẽ proportione magnitudines dicũtur eſſe, prima ad ſecundam, & tertia ad quartam, cùm primæ & tertiæ æquè multiplicia, à ſecundæ & quartæ æquè multiplicibus,

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qualiſcunque ſit hæc multiplicatio, vtrunque ab vtroque; vel vnà deficiunt, vel vnà æqualia ſunt, vel vnà excedunt; ſi ea ſumantur, quæ inter ſe reſpondent.

7.

Eandem autem proportionem habentes magnitudines, proportionales vocentur.

8.

Cùm verò æquè multiplicium, multiplex primæ magnitudinis exceſſerit multiplicē ſecundæ; at multiplex tertiæ non exceſſerit multiplicem quartæ; tunc prima ad ſecundam, maiorem proportionem habere dicetur, quàm tertia ad quartam.

9.

Proportinoalitas autem in tribus terminis paucíſsimis conſiſtit.

10.

Cùm autem tres magnitudines proportionales, fuerint, prima ad tertiam, duplicatam proportionem habere dicitur eius, quam habet ad ſecundam. At cùm quatuor magnitudines proportionales fuerint, prima ad quartam, triplicatam proportionem habere dicitur eius, quam habet ad ſecundam: & ſemper deinceps, vno amplius, quamdiu proportio extiterit.

11.

Homologæ, ſeu ſimiles proportione magnitudines dicuntur, antecedentes quidem ante-

antecedentibus, conſequentes verò conſequentibus.

12.

Alterna proportio, eſt ſumptio antecedentis ad antecedentem,&conſequentis ad conſequenrem.

13.

Inuerſa proportio, eſt ſumptio conſequentis,ceu antecedentis,ad antecedentem,velut ad ipſam conſequentem.

14.

Compoſitio proportionis, eſt ſumptio antecedentis cum conſequente, ceu vnius ad ipſam conſequentem.

15.

Diuiſio proportionis,eſt ſumptio exceſſus, quo conſequentem ſuperat antecedens, ad ipſam conſequentem.

16.

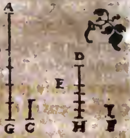
Conuerſio proportionis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſam conſequentem.

17.

Ex æqualitate proportio eſt, ſi plures duabus ſint magnitudines,& his aliæ multitudine pares, quæ binæ ſumantur, & in eadem proportione: quum vt in primis magnitudinibus prima ad vltimam,ſic& in ſecundis magnitudinibus prima ad vltimam ſeſe habuerit.

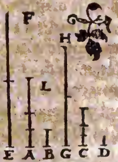
Theorema 2. Propopoſitio 2.

Si prima ſecundæ æquè fuerit multiplex, atque tertia quartæ; fuerit autem & quinta ſecundæ æquè multiplex, atque ſexta quartæ: erit & compoſita prima cũ quinta, ſecundæ æquè multiplex; atq; tertia cum ſexta, quartæ.



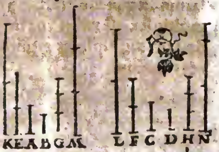
Theorema 3. Propoſitio 3.

Si ſit prima ſecundæ æquè multiplex, atque tertia quartæ; ſumantur autem æquè multiplices primæ, & tertiæ; erit & ex æquo, ſumptarum vtraque vtriuſque æquè multiplex; altera quidem ſecundæ, altera autem quartæ.



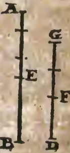
Theorema 4. Propoſitio 4.

Si prima ad ſecundã, eandem habuerit proportionẽ, & tertia ad quartam: etiam æquè multiplices primę & tertię, ad ęquè multiplices ſecundæ & quartæ, iuxta quãmuis multiplicationẽ, eandem



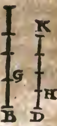
dem habebunt propoſitionem; ſi, prout inter ſe reſpondent, ita ſumptæ fuerint.

Theorema 5. Propoſitio 5.



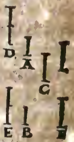
Si magnitudo magnitudinis æquè fuerit multiplex, atque ablata ablatæ: etiam reliqua reliquæ ita multiplex erit, vt tota totius.

Theorema 6. Propoſitio 6.



Si duæ magnitudines, duarum magnitudinum ſint æquè multiplices, & detractæ quædam ſint earundem æquè multiplices: & reliquæ eiſdem aut æquales ſunt, aut æquè ipſarum multiplices.

Theorema 7. Propoſitio 7.

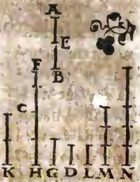


Aequales ad eandem, eandem habent rationem: & eadem ad æquales.

Theorema 8. Propoſitio 8

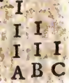
Inæqualium magnitudinum maior ad eandem

dem, maiorem proportionem habet: quàm minor: & eadem ad minorem, maiorem proportionem habet, quàm ad maiorem.



Theorema 9. Propoſitio 9.

Quæ ad eandem, eandem habent proportionem, æquales ſunt inter ſe: & ad quas eadem, eandem habet proportionem, eæ quoq; ſunt inter ſe æquales.



Theorema 10. Propoſitio 10.

Ad eandem magnitudinem proportionem habentium, quę maiorem proportionem habet, illa maior eſt, ad quam autem eadem maiorem proportionem habet, illa minor eſt.



Theorema 11. Propoſitio 11.

Quæ eidem ſunt eædem proportiones, & inter ſe ſunt eædem.



Theo-

Theorema 12. Propoſitio. 12.

Si ſint magnitudines quotcunque proportionales; quemadmodũ ſe habuerit vna antecedentium ad vnā



conſequentium; ita ſe habebunt omnes antecedentes ad omnes conſequentes.

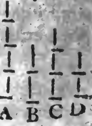
Theorema 13. Propoſitio 13.

Si prima ad ſecundã, eandem habuerit proportionẽ, quam tertia ad quartã; tertia verò ad quartam maiorẽ proportionẽ habuerit, quàm quinta ad ſextam: prima quoq; ad ſecundã maiorẽ proportionẽ habebit, quàm quinta ad ſextam.



Theorema 14. Propoſitio 14.

Si prima ad ſecundam eandem habuerit proportionẽ, quã tertia ad quartã: prima verò quàm tertia maior fuerit, erit & ſecunda maior, quàm quarta. Quòd ſi prima fuerit æqualis tertiæ,

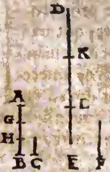


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erit ſecunda æqualis quartæ: ſi verò minor, & minor erit.

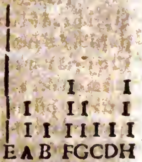
Theorema 15. Propoſitio 15.

Partes cum pariter multiplicibus in eadem ſunt proportione, ſi prout ſibi mutuò reſpondent, ita ſumantur.



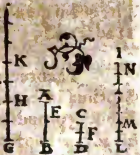
Theorema 16. Propoſitio 16.

Si quatuor magnitudines proportionales fuerint, & viciſſim proportionales erunt.

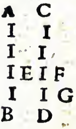


Theorema 17. Propoſitio 17.

Si compoſitæ magnitudines proportionales fuerint, hæ quoq; diuiſæ proportionales erunt.

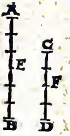


Theorema 18. Propoſitio 18.



Si diuiſæ magnitudines ſint proportionales, hæ quoque compoſitæ proportionales erunt.

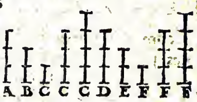
Theorema 19. Propoſitio 19.



Si quemadmodum totum ad totum, ita ablatum ſe habuerit ad ablatum: & reliquum ad reliquum, vt totum ad totum, ſe habebit.

Theorema 20. Propoſitio 20.

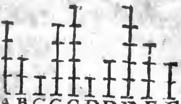
Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur; ex æquo autem prima quàm tertia maior fuerit; erit & quarta, quam ſexta, maior. Quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, quoque minor erit.



Theo-

Theorema 21. Propoſitio 21.

Si ſint tres magnitudines, & aliæ ipſis æquales numero, quæ binæ, & in eadem proportione ſumantur, fueritque perturbata earum proportio: ex æquo autẽ prima, quã tertia, maior fuerit; erit & quarta, quàm ſexta, maior: quòd ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hæc quoque minor erit.



Theorema 22. Propoſitio 22.

Si ſint quodcunque magnitudines, & aliæ ipſis æquales numero, quæ binæ in eadẽ proportione ſumantur: & ex æqualitate in eadem proportione erunt.



Theorema 23. Propoſitio 23.

Si ſint tres magnitudines, aliæque ipſis æqua

les numero, quæ binæ in eadem proportione ſumantur; fuerit autẽ perturbata earum proportio: Etiam ex æqualitate in eadem proportione erunt.



Theorema 24. Propoſitio 24.

Si prima ad ſecundam, eandem habuerit proportionem, quam tertia ad quartam; habuerit autem & quinta ad ſecundam, eandem proportionem, quàm ſexta ad quartam: Etiam compoſita prima cum quinta, ad ſecundam eandem habebit proportionem, quam tertia cum ſexta, ad quartam.



Theorema 25. Propoſitio 25.

Si quatuor magnitudines proportionales fuerint; maxima, & minima reliquis duabus maiores erunt.



Theo-

Theorema 26. Propopoſitio 26.

Si prima ad ſecundam, maiorem habuerit proportionem, quàm tertia ad quartam: habebit couertendo ſecunda ad primam, minorem proportionem, quàm quarta ad tertiam.

Theorema 27. Propoſitio 27.

Si prima ad ſecundam, habuerit maiorem proportionem, quàm tertia ad quartam: habebit quoq; viciſsim prima ad tertiam, maiorem proportionem, quàm ſecunda ad quartam.

Theorema 28. Propoſitio. 28.

Si prima ad ſecundam habuertt maiorem proportionem, quàm tertia ad quartam: habebit quoque compoſita prima cum ſecunda, ad ſecundam, maiorem proportionem, quàm compoſita tertia cũ quarta, ad quartam.

Theorema 29. Propoſitio 29.

Si compoſita prima cum ſecunda, ad ſecundam, maiorem habuerit proportionem quã

compoſita tertia cum quarta, ad quartam: Habebit quoq; diuidando prima ad ſecundam, maiorem proportionem, quam tertia, ad quartam.

Theorema 30. Propoſitio 30.

Si compoſita prima cum ſecunda, ad ſecundam habuerit maiorem proportionem, quàm compoſita tertia cum quarta, ad quartã: Habebit quoq; per conuerſionem proportionis, prima cum ſecunda, ad primam, minorem proportionem, quàm tertia cum quarta, ad tertiam.

Theorema 31. Propoſitio 31.

Si ſint tres magnitudines, & aliæ ipſis æquales numero; ſitque maior proportio primæ priorum ad ſecundam, quàm primæ poſteriorum ad ſecundam; Item ſecundæ priorem ad tertiam maior quàm ſecundæ poſteriorum ad tertiam: Erit quoque ex æqualitate maior proportio primæ priorum ad tertiam, quàm primæ poſteriorum ad tertiam.

Theorema 32. Propoſitio 32.

Si ſint tres magnitudines, & aliæ ipſis æquales numero; ſitque maior proportio primæ priorum ad ſecundam, quàm ſecundę poſteriorum

riorum ad tertiam;Item ſecundæ priorum ad tertiam maior, quàm primæ poſteriorũ ad ſecundam: Erit quoque ex æqualitate, maior proportio primæ priorum ad tertiam, quàm primæ poſteriorum ad tertiam.

Theorema 33. Propoſitio 33.

Si fuerit maior proportio totius ad totum, quàm ablati ad ablatum: Erit & reliqui ad reliquum, maior proportio, quàm totius ad totum.

Theorema 34. Propoſitio 34.

Si ſint quotcunque magnitudines, & aliæ ipſis æquales numero; ſitque maior proportio primæ priorum ad primam poſteriorum, quàm ſecũdę ad ſecundam; & hæc maior, quàm tertiæ ad quartam, & ſic deinceps: Habebunt omnes priores ſimul, ad omnes poſteriores ſimul, maiorem proportionem quàm omnes priores, relictâ primâ, ad omnes poſteriores, relictâ quoque primâ; minorem autem, quàm prima priorum ad primam poſteriorum; maiorem denique etiã, quàm vltima priorum ad vltimam poſteriorum.

FINIS ELEMENTI V.

EVCLI-

EVCLIDIS ELEMENTVM SEXTVM.

DEFINITIONES.

1.

Similes figuræ rectilineæ ſunt, quæ & angulos ſingulos ſingulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

2.

Reciprocæ autem figuræ ſunt, cùm in vtraque figura, antecedentes, & conſequentes proportionum termini fuerint.

3.

Secundum extremam, & mediam proportionem recta linea ſecta eſſe dicitur, cùm vt tota ad maius ſegmentum, ita maius ad minus ſe habuerit.

4.

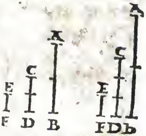
Altitudo cuiuſque figuræ eſt linea perpendicularis à vertice ad baſin deducta.

5. Ra-

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5.

Proportio proportionibus componi dicitur, cũ proportionum qtãtitates inter ſe multiplicatæ, aliquã effecerint proportionem.



6.

Parallelogrammum ſecundum aliquam lineam applicatum, deficere dicitur parallelogrammo, quando non occupat totam lineam: Excedere vero, quando occupat maiorem lineam, quàm ſit ea, ſecundum quam applicatur: Ita tamen, vt parallelogrammũ deficiens, aut excadens, eandem habeat altitudinem cum parallelogrammo applicato, conſtituatque cum eo totum vnum parallelogrammum.

Theorema 1. Propoſitio 1.

Triangula, & parallelogramma, quorum eadem fuerit altitudo, ita ſe habent inter ſe, vt baſes.



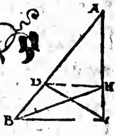
Theorema 2. Propoſitio 2.

Si ad vnum trianguli latus parallela ducta fuerit recta quædam linea:

Hæc proportionaliter ſecabit ipſius tri-

 angu-

anguli latera. Et ſi trianguli latera proportionaliter ſecta fuerint; quæ ad ſectiones adiuncta fuerit recta linea, erit ad reliquum ipſius trianguli latus parallela.



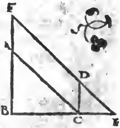
Theorema 3. Propoſitio 3.

Si trianguli angulus bifariam ſectus ſit; ſecans autem angulum recta linea ſecuerit & baſin: baſis ſegmenta, eandē habebunt proportionē, quā reliqua ipſius trianguli latera: Et ſi baſis ſegmenta eandem habeant proportionem quam reliqua ipſius trianguli latera; recta linea, quæ à vertice ad ſectionem producitur, bifariam ſecat trianguli ipſius angulum.



Theorema 4. Propoſitio 4.

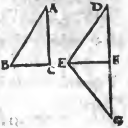
Aequiangulorum triangulorum proportionalia ſunt latera, quæ circum ęquales angulos, & homologa ſunt latera, quæ ęqualibus angulis ſubtenduntur.



Theo-

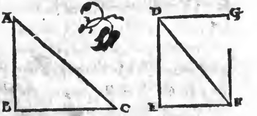
Theorema 5. Propoſitio. 5.

Si duo triangulo, latera proportionalia habeant, æquiangula erunt triangula, & æquales habebunt eos angulos, ſub quibus homologa latera ſubtenduntur.



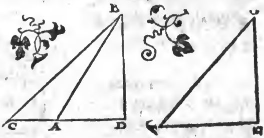
Theorema 6. Propoſitio 6.

Si duo triangula vnum angulum vni angulo æqualé, & circum æquales angulos latera proportionalia habuerint; æquiangula erunt triangula, æqualeſque habebunt angulos, ſub quibus homologa latera ſubtenduntur.



Theorema 7. Propoſitio 7.

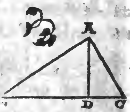
Si duo triangula vnum angulum vni angulo æqualé circũ autẽ alios angulos latera proportionalia habent; reliquorum



vero ſimul vtrunque aut minorem, aut non minorem recto: æquiangula erunt triangula, & æquales habebunt eos angulos, circum quos proportionelia ſunt latera.

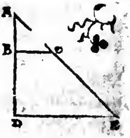
Theorema 8. Propoſitio 8.

Si in triangulo rectangulo, ab angulo recto in baſin perpendicularis ducta ſit; quæ ad perpendicularem triangula, tum toti triangulo, tum ipſa inter ſe ſimilia ſunt.



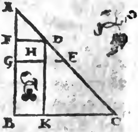
Problema 1. Propoſitio. 9.

A data recta linea imperatam partem auferre.



Problema 2. Propoſitio 10.

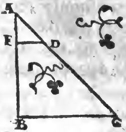
Datam rectam lineam inſectam ſimiliter ſecare, vt data altera recta ſecta fuerit.



Pro-

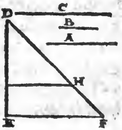
Problema 3. Propoſitio. 11.

Duabus datis rectis lineis, tertiam proportionalem adinuenire.



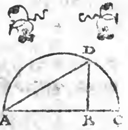
Problema 4. Propoſitio 12.

Tribus datis rectis lineis quartam proportionalẽ adinuenire.



Problema 5. Propoſitio 13.

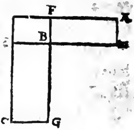
Duabus datis rectis lineis, mediam proportionalem adinuenire.



Theorema 9. Propoſitio 14.

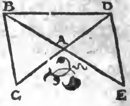
Aequalium, & vnum vni æqualem habentium angulum, parallelogrammorum reciproca ſunt latera, quæ circum æquales angulos; Et quorum parallelogrammorũ vnum angu-

angulum vni angulo æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.



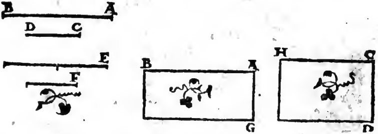
Theorema 10. Propoſitio 15.

Aequalium, & vnum angulum vni ęqualem habentium triangulorum reciproca ſunt latera, quæ circum æquales angulos; Et quorum triangulorum vnum angulum vni æqualem habentium, reciproca ſunt latera, quæ circũ æquales angulos, illa ſunt ęquales.



Theorema 11. Propoſitio 16.

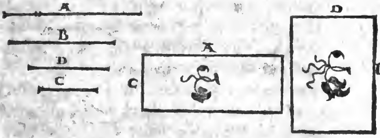
Si quatuor rectæ lineæ proportionales fuerint, quod ſub extremis comprænhenditur rectangulum, ęquale eſt ei, quod ſub medijs com-



comprehenditur, rectangulo. Et ſub extremis comprehenſum rectangulum æquale fuerit ei, quod ſub medijs continetur, rectã-gulo, illæ quatuor rectæ lineę proportionales erunt.

Theorema 12. Propoſitio 17.

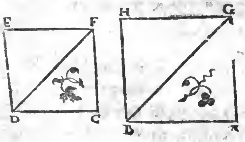
Si tres rectæ lineę ſint proportionales; quod ſub extremis comprehẽditur rectangulum



æquale ſit ei, quod à media deſcribitur, quadrato: Et ſi ſub extremis comprehẽſum rectangulum æquale ſit ei, quòd à media deſcribitur, quadrato; illę tres rectę lineæ proportionales erant.

Theorema 6. Propoſitio 16.

A data recta linea, dato recti lineo ſimile, ſimiliterq́; poſitum rectilineum deſcribere.

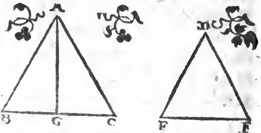


Theo-

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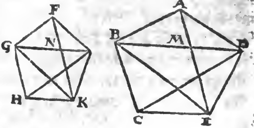
Theorema 13. Propoſitio 19.

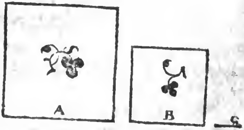
Similia triangula inter ſe ſunt in duplicata proportione laterum homologorum.

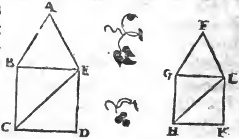


Theorema 14. Propoſitio 20.

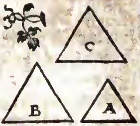
Similia polygona in ſimilia triangula diuiduntur, &numero ęqualia, &homologa totis. Et polygona duplicatam habẽt eam inter ſe proportionem, quam latꝰ homologum ad homologum latus.







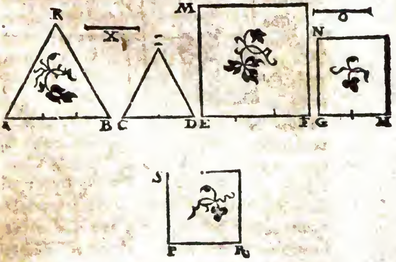
Theorema 15. Propoſitio 21.



Quæ eidem rectilineo ſunt ſimilia, & inter ſe ſunt ſimilia.

Theorema 16. Propoſitio 22.

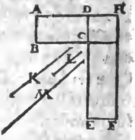
Si quatuor rectæ lineæ proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterque deſcripta proportionalia erunt. Et ſi à rectis lineis ſimilia ſimiliterque deſcripta rectilinea proportionalia fuerint; ipſæ etiam rectæ lineæ proportionales erunt.



Prop.

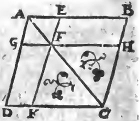
Theorema 17. Propoſitio 23.

Aequiangula parallelogramma inter ſe proportionem habent eam, quæ ex lateribus componitur.

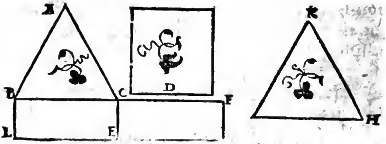


Theorema 18. Propoſitio 24.

In omni parallelogrammo, quæ circa diametrũ ſunt parallelogramma, & toti, & inter ſe ſunt ſimilia.



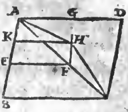
Problema 7. Propoſitio 25.



Dato recti lineo ſimile: ſimiliterque poſitum; & alteri dato æquale idem conſtituere.

Theo-

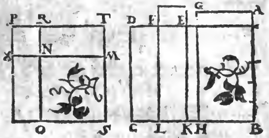
Theorema 19. Propoſitio 26.



Si à parallelogrammo parallelogrammum ablatum ſit, & ſimile toti, & ſimiliter poſitum; communem cum eo habens angulum; hoc circum eandem cum toto diametrum conſiſtit.

Theorema 20. Propoſitio 27.

Omnium parallelogrammorum ſecũdum eandem rectam lineam applicatorum, deficientiumq́; figuris parallelogrãmis ſimilibus, ſi-

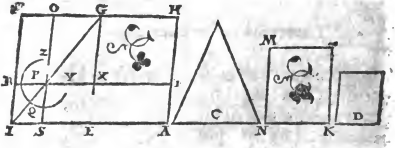


militerque poſitis, ei, quod à dimidia deſcribitur; maximum, id eſt, quod ad dimidiam applicatur, parallelogrammum ſimile exiſtens defectui.

Problema 8. Propoſitio 28.

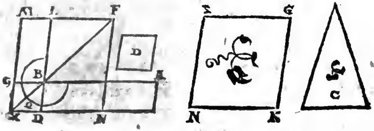
Ad datam lineam rectam, dato rectilineo æquale parallelogrammum applicare, deficiens figura parallelogramma, quæ ſimilis

ſit alteri rectilineo dato. Oportet autem datum rectilineum, cui æquale applicandum eſt, non maius eſſe eo, quod ad dimidiam applicatur, cùm ſimiles ſint defectus & eius, quod à dimidia deſcribitur, & eius, cui ſimile deeſſe debet.



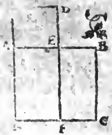
Problema 9. Propoſitio 29.

Ad datam rectam lineam, dato rectilineo æquale parallelogrammum applicare, excedens figura parallelogramma, quę ſimilis ſit parallelogrammo alteri dato.



Pro-

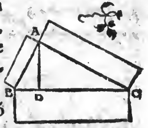
Problema 10. Propoſitio 30.



Propoſitam rectam lineam terminatam, extrema ac media ratione (proportione:) ſecare.

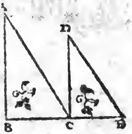
Theorema 21. Propoſitio 31.

In rectangulis triangulis, figura quæuis à latere rectum angulum ſubtendente deſcripta, æqualis eſt figuris, quæ priori illi ſimiles, & ſimiliter poſitæ, à lateribus rectum angulum cõtinentibus deſcribũtur.



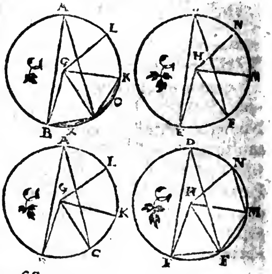
Theorema 22. Propoſitio 32.

Si duo triangula, quæ duo latera duobus lateribus proportionalia habeant, ſecundum vnum angulum compoſita fuerint, ita vt homologa eorum latera ſint etiam parallela; tum reliqua illorum triangulorum latera in rectam lineam collocata reperientur.



Theorema 23. Propoſitio 33.

In æqualibus circulis, anguli eandem habent rationem, cum ipſis peripherijs, in quibus inſiſtũt, ſiue ad centra, ſiue ad peripherias cõſtituti, illis inſiſtant peripherijs. Inſuper verò & ſectores, quippe qui ad centra conſiſtunt.



FINIS ELEMENTI VI.

EVCLI-

EVCLIDIS ELEMENTVM SEPTIMVM.

DEFINITIONES.

1.

VNitas eſt, ſecundum quam vnum, quodque eorum, quæ ſunt, vnum dicitur.

2.

Numerus autem, ex vnitatibus compoſita multitudo.

3.

Pars eſt numerus numeri, minor maioris, cùm minor metitur maiorem.

4.

Partes autem, cùm non metitur.

5.

Multiplex verò, maior minoris, cùm maiorem metitur minor.

6.

Par numerus eſt, qui bifariam diuiditur.

7.

Impar verò, qui bifariam non diuiditur: vel, qui vnitate differt à pari.

8

Pariter par numerus eſt, quem par numerus metitur per numerum parem.

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9.

Pariter autem impar eſt, quem par numer[us] metitur per numerum imparem.

10.

Impariter verò impar numerus eſt, quem impar numerus metitur per numerum imparem.

11.

Primus numerus eſt, quem vnitas ſola metitur.

12.

Primi inter ſe numeri ſunt, quos ſola vnitas, menſura communis, metitur.

13.

Compoſitus numerus eſt, quem numerus quiſpiam metitur.

14.

Compoſiti autẽ inter ſe numeri ſunt, quos numerus aliquis, menſura communis, metitur.

15.

Numerus numerum multiplicare dicitur, cùm toties compoſitus fuerit is, qui multiplicatur, quot ſunt in ipſo multiplicante vnitates; & procreatus fuerit aliquis.

16.

Cùm autem duo numeri mutuò ſeſe multiplicantes quempiam faciunt, qui factus erit, planus appellabitur. Qui verò numeri mutuò ſeſe multiplicarint, illius latera dicentur.

Cùm

17.

Cùm verò tres numeri mutuò ſeſe multiplicantes quempiam faciunt, qui procreatus erit, ſolidus appellabitur, qui autem numeri mutuò ſeſe multiplicarint, illius latera dicentur.

18.

Quadratus numerus eſt, qui æqualiter ęqualis: vel, qui à duabus æqualibus numeris cõtinetur.

19.

Cubus verò, qui æqualiter æquali æqualiter: vel, qui à tribus æqualibus numeris continetur.

20.

Numeri proportionales ſunt, cùm primus ſecundi, & tertius quarti, æquè multiplex eſt; vel eadem pars, vel eædem partes.

21.

Similes plani, & ſolidi numeri ſunt, qui proportionalia habent latera.

22.

Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

23.

Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

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24.

Proportio numerorum eſt habitudo quædam vnus numeri & alterum, ſecundum quod illius eſt multiplex, vel pars, partesue.

25.

Termini, ſiue radices proportionis dicuntur duo numero, quibus in eadem proportione minores ſumi nequeunt.

26.

Cùm tres numeri proportionales fuerint; Primus ad tertium, duplicatam proportionem habere dicitur eius, quam habet ad ſecundum. At cùm quatuor numeri proportionales fuerint; primus ad quartum, triplicatam proportionem habere dicitur eius, quam habet ad ſecundum. Et ſuper deinceps vno amplius, quamdiu proportio extiterit.

27.

Quotlibet numerus ordine poſitis, proportio, primi ad vltimum componi dicitur ex proportionibus primi ad ſecundum, & ſecundi ad tertium, & tertij ad quartum, & ita deinceps, donec extiterit proportio.

Poſtulata, ſiue petitiones.

1.

Poſtuletur, cuilibet numero, quotlibet poſſe ſumi æquales, vel multiplices.

2.

Quolibet numero, ſumi poſſe maiorem.

Axio-

Axiomata,ſiue pronunciata.

1.

Qui numeri æqualium numerorum, vel eiuſdem, æquè multiplices ſunt, inter ſe ſunt æquales,

2.

Quorum idem numerus æquè multiplex eſt, vel æquè multiplices ſunt æquales, inter ſe æquales ſunt.

3.

Qui numeri æqualium numerorum, vel eiuſdem, eadam pars, partes fuerint, inter ſe æquales ſunt.

4.

Quorum idem numerus, vel æquales, eadē pars, vel eædem partes fuerint, æquales inter ſe ſunt.

5.

Vnitas omnem numerum per vnitates, quæ in ipſo ſunt, hoc eſt, per ipſummet numerū, metitur.

6.

Omnis numerus ſeipſum metitur per vnitatem.

7.

Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum; multiplicatus autem eundem per multiplicantem.

8.

Si numerus numerum metiatur, vt ille, per quem metitur, eundem metietur per eas, quæ in metiente ſunt, vnitates, hoc eſt, per ipſum numerum metientem.

9.

Si numerus numerum metiens, multiplicet eum, per quem metitur; vel ab eo multiplicatur, illum, quem metitur, producat.

10.

Numerus quotcunque numeros metiens, compoſitum quoque ex ipſis metitur.

11.

Numerus quemcunq; numerum metiens, metitur quoque omnem numerum, quem ille metitur.

12.

Numerus metiens totum, & ablatum, metitur & reliquum.

Theorema 1. Propoſitio 1.

Si Duobus numeris inæqualibus propoſitis, detrahatur ſemper minor de maiore, alterna quadam detractione; neque reliquus vnquam metiatur præcedentem, quo ad aſſumpta ſit vnitas: qui principiò propoſiti ſunt numeri, primi inter ſe erunt.

A		
H		
:		C
F		:
:		G
:	:	:
B	D	E

Pro-

Problema 1. Propoſitio 2.

Duobns numeris datis non primis inter ſe, maximam eorum communem menſuram reperire.

A
A : C
E
: E E
: : : :
B D B D
: : : : :

Problema 2. Propoſitio 3.

A B C D E
8 6 4 2 3

Tribus numeris datis non primis inter ſe, maximam eorum communem menſuram reperire.

: : : : : :
A B C D E F
18 13 8 6 2 3

Theorema 2. Propoſitio 4.

C
:
F

Omnis numerus cuiuſq; numeri, minor maioris, aut pars eſt, aut partes.

C C :
E
: :
: : : : : :
A B B B D
12 7 6 9 3

Theorema 3. Propoſitio 5.

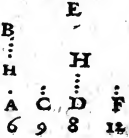
Si numerus numeri pars fuerit, & alter alterius eadē pars; & ſimul vterque vtriuſque ſimul eadem pars erit, quæ vnus eſt vnius.

C
: F
: :
G H
: : : :
: : : :
A B D C
6 2 1 ſl 8

Theo-

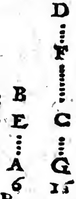
Theorema 4 Propoſitio 6.

Si numerus ſit numeri partes, & alter alterius eædem partes; & ſimul vterque vtriuſque ſimul eædem partes erunt, quæ ſunt vnus vnius.



Theorema 5. Propoſitio 7.

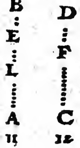
Si numerus numeri eadem ſit pars quæ detractus detracti; & reliquus reliqui eadem pars erit, quæ totus eſt totius.



Theorema 6. Propoſitio 8.

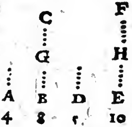
Si numerus numeri eædem ſint partes, quæ detractus detracti; & reliquus reliqui eædem partes erunt, quæ ſunt totus totius.

G.M.K.N.H.



Theorema 7. Propoſitio 9.

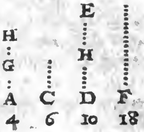
Si numerus numeri pars ſit, & alter alterius eadem pars: & viciſsim, quæ pars eſt, vel partes primus tertij, eadem pars erit vel eædem partes, & ſecundus 4 quarti.



Theo-

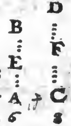
Theorema 8. Propoſitio 10.

Si numerus numeri partes ſint, & alter alterius eadem partes: etiam viciſsim, quæ ſunt partes, aut pars primus tertij, eędẽ partes erunt, vel pars & ſecundus quarti.



Theorema 9. Propoſitio 11.

Si quemadmodum ſe habet totus ad totum, ita detractus ad detractum: & reliquis ad reliquum ita ſe habebit, vt totus ad totum.



Theorema 10. Propoſitio 12.

Si ſint quotcunque numeri proportionales; quemadmodum ſe habet vnus antecedentium ad vnum conſequentium, ita ſe habebunt omnes antecedentes ad omnes conſequentes.

:	:	:	:
A	B	C	D
9	6	3	2

Theorema 11. Propoſitio 13.

Si quatuor numeri ſint proportionales; & viciſſim proportionales erũt.

:	:	:	:
A	B	C	D
12	4	9	3

Theorema 12. Propoſitio 14.

Si ſint quotcunque numeri, & alij illis ęquales multitudi-

:	:	:	:	:	:
A	B	C	D	E	F
12	6	3	8	4	2

ne,

ne, qui bini ſumantur, & in eadem proportione, etiam ex æqualitate in eadem proportione erunt.

Theorema 13. Propoſitio 15.

Si vnitas numerum quēpiam metiatur, alter verò numerus alium quendam numerum æquè metiatur; & viciſsim vnitas tertium numerum ęquè metietur, atq; ſecūdus quartum.

			F
	C		L
	H		K
	G		E
A	B		D
1	3	2	6

Theorema 14. Propoſitio 16.

Si duo numeri mutuò ſeſe multiplicātes faciant aliquos; qui ex illis geniti fuerint, inter ſe æquales erunt.

E	A	B	C	D
1	2	4	8	8

Theorema 15. Propoſitio 17.

Si numerus duos numeros multiplicans faciat aliquos; qui ex illis procreati erunt, eandē proportionem inter ſe habebunt, quam multiplicati.

I	A	B	C	D	E
1	3	4	5	12	15

Theorema 16. Propoſitio 18.

Si duo numeri numerum quempiam multiplicantes, faciant aliquos; geniti ex illis eandem habebunt proportionem, quam qui illum multiplicarunt.

A	B	C	D	E
4	5	3	12	15

Theo-

Theorema 17. Propoſitio 19.

Si quatuor numeri ſint proportionales; quod ex primo, & quarto fit, numerus, æqualis erit ei, qui ex ſecundo & tertio, fit, numero: Et ſi, qui ex primo & quarto ſit numerus, æqualis ſit ei, qui ex ſecundo & tertio, fit, numero; illi quatuor numeri proportionales erunt.

:	:	:	:	:	:	:
A	B	C	D	E	F	G
6	4	3	2	12	12	18

Theorema 18. Propoſitio 20.

Si tres numeri ſint proportionales; qui ab extremis continetur, æqualis eſt ei, qui à medio efficitur: Et ſi, qui ab extremis continetur, æqualis ſit ei, qui à medio deſcribitur, illi tres numeri proportionales erunt.

:	:	:
A	B	C
9	6	4
:		
D		
6		

Theorema 19. Propoſitio 21.

Minimi numeri omnium qui eandem cum eis proportionem habent, æqualiter metiuntur numeros eandem cum eis proportionem habentes; maior quidem maiorem, minor verò minorem.

D	L		
:	:		
G	H		
:	:	:	:
C	E	A	B
4	3	8	6

Theo-

Theorema 20. Propopoſitio 22.

Si tres ſint numeri, & alij multitudine illis æquales, qui bini ſumantur & in eadem ratione; ſit autem perturbata eorum proportio; etiam ex æqualitate in eadem proportione erunt.

A	B	C	D	E	F
6	4	3	12	8	6

Theorema 21. Propoſitio 23.

Primi inter ſe numeri, minimi ſunt omniũ eandem cum eis proportionem habentium.

A	B	E	C	D
5	6	2	4	3

Theorema 22. Propoſitio 24.

Minimi numeri omnium eandem cum eis proportionem habentium, primi ſunt inter ſe.

A	B	C	D	E
7	6	4	3	2

Theorema 23. Propoſitio. 25.

Si duo numeri ſint primi inter ſe, qui alterutrum eorum metitur, numerus, is ad reliquũ primus erit.

A	B	C	D
6	7	3	4

Theorema 24. Propoſitio. 26.

Si duo numeri ad quẽpiam numerum primi ſint, ad eundem primus is quoque futurus eſt, qui ab illis productus fuerit.

B	A	C	D	E	F
5	5	5	5	3	2

Theo-

Theorema 25. Propoſitio 27.

Si duo numeri primi ſint inter ſe, qui ab vno eorum gignitur, ad reliquum primus erit.

B	C	D
A		
7	6	3

Theorema 26. Propoſitio 28.

Si duo numeri ad duos numeros ambo ad vtrunque primi ſint; & qui ex eis gignentur, primi inter ſe erunt.

A	B	E	C	D	F
3	5	15	2	4	8

Theorema 27. Propoſitio 29.

Si duo numeri primi ſint inter ſe, & multiplicans vterque ſeipſum procreet aliquem; qui ex ijs producti fuerint, primi inter ſe erunt. Quòd ſi numeri initiò propoſiti multiplicantes eos, qui producti ſunt, effecerint aliquos; hi quoque inter ſe primi erunt; & circa extremos idem hoc ſemper eueniet.

A	C	E	B	D	F
3	6	27	4	16	63

Theorema 28. Propoſitio 30.

Si duo numeri primi ſint inter ſe, etiam ſimul vterque ad vtrunque illorum primus erit. Et ſimul vterque ad vnum aliquem eorum primus ſit, etiam qui initio poſiti ſunt numeri, primi inter ſe erunt.

C		
A	B	D
7	5	4

Theorema 29. Propoſitio 31.

Omnis primus numerus ad omnem numerum, quem non metitur, primus eſt.

:	:	:
A	B	C
7	105	

Theorema 30. Propoſitio 32.

Si duo numeri ſeſe mutuò multiplicantes faciant aliquem; hunc autem ex ipſis productum metiatur primus quidam numerus: is alterum etiam eorum, qui initiò poſiti erant, metietur.

:	:	:	:	:
A	B	C	D	E
3	6	12	3	4

Theorema 31. Propoſitio 33.

Omnem compoſitum numerum aliquis primus metietur.

:	:	:
A	B	C
27	9	3

Theorema 32. Propoſitio 34.

Omnis numerus aut primus eſt, aut eum aliquis primus metitur.

:	:	:
A	A	
3	6	3

Problema 3. Propoſitio 35.

Numeris datis quotcunque, reperire minimos omnium, qui eandem cum illis proportionem habeant.

:	:	:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K	I	M
6	8	12	2	3	4	6	2	3	4	3

Pro-

Problema 4. Propoſitio 36.

B				
A	C	D	E	F
7	12	8	4	5

Duobus numeris datis, reperire, quẽ illi minimum metiantur, numerum

A				
F	E	C	D	G	H
6	9	12	0	2	3

Theorema 33. Propoſitio 37.

Si duo numeri numerum quempiam metiantur; & minimus, quem illi metiuntur, eundem metietur.

			F
A	B	E	C
2	3	6	12

Problema 5. Propoſitio. 38.

Tribus numeris datis, reperire quem minimum numerum illi metiantur.

A	B	C	D	E	
3	4	6	12	8	
A	B	C	D	E	F
3	6	8	12	24	16

Theorema 34. Propoſitio 39.

Si numerum quiſpiam numerus metiatur, menſus partem habebit metienti cognominem.

A	B	C	D
12	4	3	1

Theorema 35. Propoſitio 40.

Si numerus partem habuerit quamlibet, illum metietur numerus parti cognominis.

A	B	C	D
8	4	2	1

Problema 6. Propoſitio. 41.

Numerum reperire, qui minimus cùm ſit, datas habeat partes.

A	B	C	G	H
2	3	4	12	10

FINIS ELEMENTI VII.

EVCLI-

EVCLIDIS ELEMENTVM OCTAVVM.

DEFINITIONES.

Theorema 1. Propoſitio 1.

SI ſint quotcunque numeri deinceps proportionales, quorum extremi ſint inter ſe, primi; ipſi minimi ſunt omniũ eandem cum eis proportionem habentium.

:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H
8	12	18	27	6	8	12	18

Problema 1. Propoſitio 2.

Numeros reperire deinceps proportionales minimos, quotcunque iuſſerit quiſpiam in data proportione.

:	:	:	:	:	:	:	:	:
A	B	C	D	E	F	G	H	K
3	4	9	12	16	27	36	49	64

Theorema 2. Propoſitio 3.
Conuerſa primæ.

Si ſint quotcunque numeri deinceps proportionales minimi habentium eandem cũ

eis proportionem; illorum extremi ſunt inter ſe primi.

A	B	C	D	E	F	G	H	K	L	M	N	O
27	16	48	64	1	4	9	12	16	27	36	48	64

Problema 2. Propoſitio 4.

Proportionibus datis quotcunque in minimis numeris, reperire numeros deinceps minimos in datis proportionibus.

A	B	C	D	E	F	H	G	K	L	N	X	M	O
3	4	2	[illegible]	4	5	6	8	12	15	4	6	10	12

Theorema 3. Propoſitio 5.

Plani numeri proportionem inter ſe habent ex lateribus compoſitam.

A	L	B	C	D	E	F	G	H	K
18	22	32	3	6	4	8	9	12	16

Theorema 4. Propoſitio 6.

Si ſint quotlibet numeri deinceps,

A	B	C	D	E	F	G	H
16	24	35	54	82	4	6	9

inceps, proportionales; primus autem ſecũdum non metiatur; neq; alius quiſpiam vllum metietur.

Theorema 5. Propoſitio 7.

Si ſint quotcunque numeri deinceps proportionales; primus autem extremum metiatur; is etiam ſecundum metietur.

A B C D
4 6 12 24

Theorema 6. Propoſitio 8.

Si inter duos numeros medij continua proportione indicant numeri; quot inter eos medij continua proportione incidunt numeri, totidem & inter alios eandem cum illis habentes proportionem medij continua proportione incident.

A C D B G H K L C M N F
4 9 27 81 1 3 9 27 2 6 18 54

Theorema 7. Propoſitio 9.

Si duo numeri ſunt inter ſe primi, & inter eos medij continua proportione incidant numeri, quot inter eos medij continua proportione incidunt numeri, totidem & inter vtrunque eorum, ac vnitatem deinceps medij continua proportione incident.

A	M	H	E	F	N	C	K	X	G	D	L	O	B
27	27	9	36	3	36	1	12	43	4	48	16	64	64

Theorema 8. Propoſitio 10.

Si inter duos numeros, & vnitatem, continuè proportionales incidant numeri; quot inter vtrunque ipſorum, & vnitatem, deinceps medij continua ꝓportione incidunt numeri; totidẽ & inter illos medij continua proportione incident.

A 27	E 9	D 3	K 36	H 12	C 1	
			L 48	F 4	G 16	B 64

Theorema 9. Propoſitio 11.

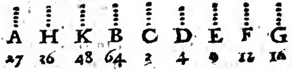
Duorum quadratorum numerorum vnus medius proportionalis eſt numerus: & quadratus ad quadratum duplicatam habet lateris ad latus proportionem.

A	C	E	D	B
9	3	12	4	16

Theorema 10. Propoſitio 12.

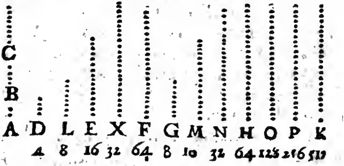
Duorum cuborum numerorum duo medij proportionales ſunt numerorum; Et cubus ad

ad cubum triplicatam habet lateris ad latus proportionem.



Theorema 11. Propoſitio 13.

Si ſint quotlibet numeri deinceps proportionales, & multiplicans quiſque ſeipſum faciat aliquos; qui ab illis producti fuerint, proportionales erunt. Et, ſi numeri primũ poſiti, ex ſuo id procreatos ductu, faciant aliquos; ipſi quoque proportionales erunt.



Theorema 12. Propoſitio 14.

Si quadratus numerus quadratum numerũ metiatur, & latus vnius metietur latus alterius.

rius. Et ſi vnius quadrati latus metiatur latus alterius, & quadratus quadratum metietur.

A	E	B	C	D
9	12	16	8	4

Theorema 13. Propoſitio 15.

Si cubus numerus cubum numerus metiatur, & latus vnius metietur alterius latus. Et ſi latus vnius cubi latus alterius metiatur, tum cubus cubum metietur.

A	H	K	B	C	D	E	F	G
8	16	28	64	2	4	4	8	16

Theorema 14. Propoſitio 16.

Si quadratus numerus quadratum numerũ non metiatur, neque latus vnius metietur alterius latus. Et ſi latus vnius quadrati non metiatur latus alterius, neque quadratus quadratum metietur.

A	B	C	D
9	16	3	4

Theorema 15. Propoſitio 17.

Si cubus numerus cubum numerum nõ metiatur; neque latus vnius latus alterius metietur. Et ſi latus cubi vnius latus alterius non metiatur, neque cubus cubum metietur.

A	B	C	D
8	27	9	11

Theo-

Theorema 16. Propoſitio 18.

Duorum ſimilium planorum numerorum vnus medius proportionalis eſt numerus;& planus ad planum duplicatam habet lateris homologi ad latus homologum proportionem.

A	G	B	C	D	E	F
12	18	27	2	6	3	9

Theorema 17.Propoſitio 19.

Duorum ſimilium numerorum ſolidorum duo medij proportionales ſunt numeri: Et ſolidus ad ſimilem ſolidum triplicatam, habet lateris homologi ad latus homologum proportionem.

A	N	X	B	C	D	E	F	G	H	K	M	L
8	12	18	27	2	2	2	3	3	3	4	6	9

Theorema 18.Propoſitio 20.

Si inter duos numeros vnus medius proportionalis incidat numerus;ſimiles plani erunt illi numeri.

A	C	B	D	E	F	G
18	24	33	3	4	6	8

Theo-

Theorema 19. Propoſitio 21.

Si inter duos numeros duo medij proportionales incidãt numeri; ſimiles ſolidi ſunt illi numeri.

A	C	D	B	E	F	G	H	K	L	M
27	36	44	64	9	12	16	3	3	3	4

Theorema 20. Propoſitio 22.

Si tres numeri deinceps ſint proportionales; primus autem ſit quadratus, & tertius quadratus erit.

A	B	D
9	15	25

Theorema 21. Propoſitio 23.

Si quatuor numeri deinceps ſint proportionales; primus autem ſi cubus, & quartus cubus erit.

A	B	C	D
8	12	18	27

Theorema 22. Propoſitio 24.

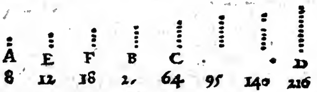
Si duo numeri eã proportionẽ habeãt inter ſe, quam quadratus numerus ad quadratum numerum, primus autem ſit quadratus, & ſecundus quadratus erit.

A		B	C		D
4	9	9	16	34	36

Theorema 23. Propoſitio 25.

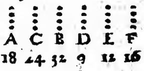
Si numeri duo proportionem inter ſe habeant,

beant, quam cubus numerus ad cubum numerum; primus autem cubus ſit, & ſecūdus cubus erit.



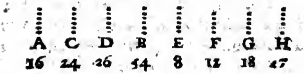
Theorema 24. Propoſitio 26.

Similes plani numeri proportionem inter ſe habent, quam quadratus numerus ad quadratum numerum.



Theorema 25. Propoſitio 27.

Similes ſolidi numeri proportionem habēt inter ſe, quam cubus numerus ad cubum numerum.



FINIS ELEMENTI VIII.

EVCLI-

EVCLIDIS ELEMENTVM NONVM.

Theorema 1. Propoſitio 1.

Si duo ſimiles plani numeri mutuò ſeſe multiplicantes, quendam procreent, productus quadratus erit.

A	E	B	D		C
4	6	9	16	24	36

Theorema 2. Propoſitio 2.

Si duo numeri mutuò ſeſe multiplicantes, quadratum faciant, illi ſimiles ſunt plani.

A		B	D		C
4	6		9	18	36

Theorema 3. Propoſitio 3.

Si cubus numerus ſeipſum multiplicãs procreet

creet aliquem, productus cubus erit.

Vnitas	D	D	A			B
	3	4	8	16	32	64

Theorema 4. Propoſitio 4.

Si cubus numerus cubũ numerum multiplicans quendam procreet, procreatus cubus erit.

A	B	D	C
8	27	64	116

Theorema 5. Propoſitio. 5.

Si cubus numerus numerum quendam multiplicans cubum procreet, & multiplicatus cubus erit.

A	B	C	D
27	64	729	27 18

Theorema 6. Propoſitio 6.

Si numerus ſeipſum multiplicans cubum procreet; & ipſe cubus erit.

A	B	C
27	729	19683

Theorema 7. Propoſitio 7.

Si compoſitus numerus quẽdam numerum multiplicans, quempiam procreet, productus ſolidus erit.

A	B	C	D	E
6	8	48	2	3

Theorema 8. Propoſitio 8.

Si ab vnitate quotlibet numeri deinceps ꝓportionales ſint: Tertius ab vnitate quadratus eſt, & vnum intermittentes omnes: Quartus aũt cubus eſt, & duobus intermiſsis omnes:

nes: Septimus verò cubus ſimul & æquadratus eſt, & quinque intermiſſis omnes

	Vni tas	A	B	C	D	E	F
		3	9	27	81	243	729

Theorema 9. Propoſitio 9.

Si ab vnitate ſint quotcunque numeri deinceps proportionales; ſit autem quadratus is, qui vnitatem ſequitur, & reliqui oẽs quadrati erũt. Quòd ſi, qui vnitatem ſequitur, cubus ſit; & reliqui omnes cubi erunt.

quadrati		cubi
531441	F	732969
59040	E	531441
6561	D	6561
723	C	6561
81	B	729
9	A	81
	o	
	vnitas.	

Theorema 10. Propoſitio 10.

Si ab vnitate numeri quotcunque proportionales ſint; non ſit autem quadratus is, qui vnitatem ſequitur, neque alius vllus quadratus

o	A	B	C	D	E	F
Vnitas.	3	9	36	81	243	729

tus erit; demptis, tertio ab vnitate ac omnibus vnum intermittentibus. Quòd ſi, qui vnitatem ſequitur, cubus non ſit: neque alius vllus cubus erit; demptis, quarto ab vnitate, ac omnibus duos intermittentibus.

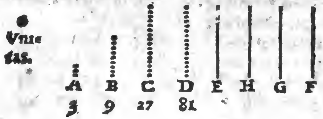
Theorema 11. Propoſitio 11.

Si ab vnitate numeri quotlibet deinceps proportionales ſint; minor maiorem metitur per quempiam eorum, qui in proportionalibus ſunt numeris.

:	:	:	:	:
A	B	C	D	E
1	2	4	8	16

Theorema 12. Propoſitio 12.

Si ab vnitate quotlibet numeri ſint proportionales; quot primorum numerorum vltimum metiuntur, totidem & eum, qui vnitati proximus eſt, metientur.

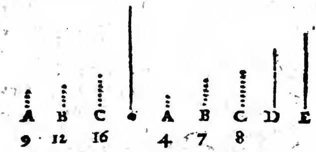


Theorema 13. Propoſitio 13.

Si ab vnitate ſint quotcunque numeri deinceps proportionales; primus autem ſit, qui vnitatem ſequitur; maximum nullus alius

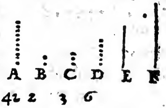
H metie-

metietur; ijs exceptis, qui in proportionalibus ſunt, numeris.



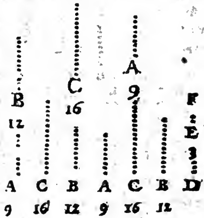
Theorema 14. Propoſitio 14.

Si minimum numerum primi aliquot numeri metiantur; nullus alius numerus primus illum metietur; ijs exceptis, qui primò metiuntur.



Theorema 15. Propoſitio 15.

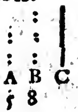
Si tres numeri deinceps proportionales ſint minimi omnium, eandem cum ipſis proportionem habentium, duo quilibet compoſiti, ad tertium primi erũt.



Theo-

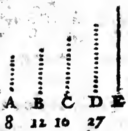
Theorema 16. Propoſitio 16.

Si duo numeri ſint inter ſe primi; non ſe habebit quemadmodum primus ad ſecundum; ita ſecundus ad quempiam alium.



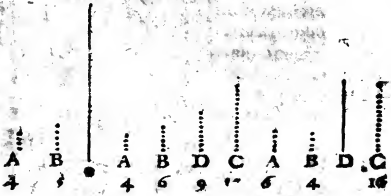
Theorema 17. Propoſitio 17.

Si ſint quotlibet numeri deinceps proportionales, quorum extremi ſint inter ſe primi; non erit quẽadmodum primus ad ſecundum, ita vltimus ad quempiam alium.



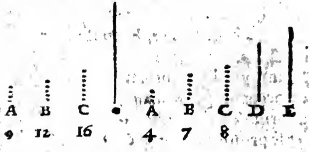
Problema 1. Propoſitio 18.

Duobus numeris datis, conſiderare an poſſit ipſis tertius inueniri proportionalis.



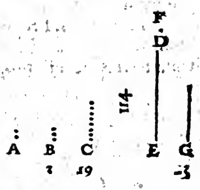
Problema 12. Propoſitio 19.

Tribus numeris datis, conſiderare, an poſſit ipſis quartus reperiri proportionalis.



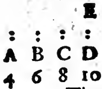
Theorema 18. Propoſitio 20.

Primi numeri plures ſunt, quacunque propoſita multitudine primorum numerorum.



Theorema 19. Propoſitio 21.

Si pares numeri quotlibet compoſiti ſint, totus eſt par.



Theo-

Theorema 20. Propoſitio 22.

Si impares numeri quotlibet compoſiti ſint; ſit autem par illorum multitudo; totus par erit.

E
A B C D
5 9 7 3

Theorema 21. Propoſitio 23.

Si impares numeri quotcunque compoſiti ſint; ſit autem impar illorum multitudo; & totus impar erit.

A B C E
5 7 8 7

Theorema 22. Propoſitio 24.

Si à pari numero par detractus ſit; & reliquus par erit.

B
A C
6 4

Theorema 23. Propoſitio 25.

Si à pari numero impar detractus ſit; & reliquus impar erit.

A C D
8 1 4

Theorema 24. Propoſitio 26.

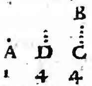
Si ab impari numero impar detractus ſit; & reliquus par erit.

B
A C D
4

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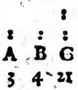
Theorema 25. Propoſitio 27.

Si ab impari numero par ablatus ſit; reliquis impar erit.



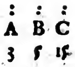
Theorema 26. Propoſitio 28.

Si impar numerus parem multiplicans, procreet quempiam; procreatus par erit.



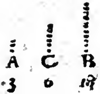
Theorema 27. Propoſitio 29.

Si impar numerus imparem numerum multiplicans, quédam procreet; procreatus impar erit.



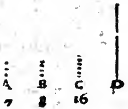
Theorema 28. Propoſitio 30.

Si impar numerus parem numerum metiatur; & illius dimidium metietur.



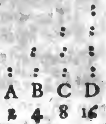
Theorema 29. Propoſitio 31.

Si impar numerus ad numerum quempiam primus ſit: & illius duplum primus erit.



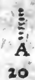
Theorema 30. Propoſitio 32.

Numerorum, qui à binario dupli ſunt, Vni-vnuſquiſque pariter tan-par eſt tantùm.



Theorema 31. Propoſitio 33.

Si numerus dimidium habeat imparem: pariter impar eſt tantùm.



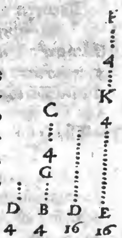
Theorema 32. Propoſitio 34.

Si par numerus neque à binario duplus ſit, neque dimidium habeat imparem: pariter par eſt, & pariter impar.



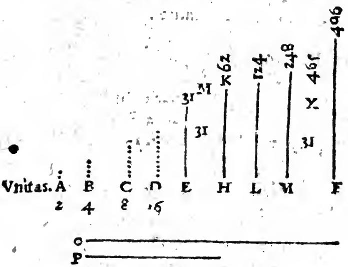
Theorema 33. Propoſitio 35.

Si ſint quotlibet numeri deinceps proportionales; detrahantur autem à ſecundo & vltimo æquales ipſi primo: Erit quemadmodum ſecundi exceſſus ad primũ, ita vltimi exceſſus ad omnes, qui vltimum antecedunt.



Theorema 34. Propoſitio 36.

Si ab vnitate numeri quotlibet deinceps expoſiti ſint in dupla proportione, quoad totus compoſitus primus factus ſit; iſque totus in vltimum multiplicatus, quempiam procreatus perfectus erit.



FINIS ELEMENTI IX.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM.

DEFINITIONES.

PRIMAE, NEMPE MAGNITVdinem ſymmetrarum.

1.

COmmenſurabiles magnitudines dicũtur illæ, quas eadem menſura metitur.

2.

Incommenſurabiles verò magnitudines dicuntur, quarum nullam menſuram communem contingit reperiri.

3.

Lineæ rectæ potentia commenſurabiles ſunt, quarum quadrata vna eadem ſuperficies, ſiue area metitur.

4.

Incommenſurabiles verò lineæ ſunt, quarum quadrata, quæ metiatur area communis, reperiri nulla poteſt.

5.

Hæc cùm ita ſint, oſtendi poteſt, quòd quãtacunque linea recta nobis proponantur, exiſtunt etiam aliæ lineæ innumerabiles eidẽ

commenſurabiles, aliæ item incommenſurabiles; hę quidem longitudine & potentia; illæ verò potentia tantùm. Vocetur igitur linea recta, quantacunq; proponatur, ῥητὴ, id eſt, rationalis.

6.

Lineæ quoque illi ῥητὴ commenſurabiles ſiue longitudine & pontentia, ſiue potentia tantùm, vocentur & ipſe ῥηταὶ, id eſt, rationales.

7.

Quæ verò lineæ ſunt incommenſurabiles illi τῇ ῥητῇ, id eſt, primo loco rationali, vocentur ἄλογοι, id eſt, irrationales.

8.

Et quadratum, quod à linea propoſita deſcribitur, quam ῥητὴν vocari volumus, vocetur ῥητὸν, id eſt, rationale.

9.

Et, quæ ſunt huic commenſurabilia, vocentur ῥητὰ, id eſt, rationalia.

10.

Quæ verò ſunt illi quadrato, ῥητῷ ſcilicet, incommenſurabilia, vocentur, ἄλογα, id eſt, ſurda, ſiue irrationalia.

11.

Et lineæ, quæ illa incommenſurabilia deſcribunt, vocentur ἄλογοι. Et quidam ſi illa incommenſurabilia fuerint quadrata, ipſa eorum altera vocabũtur ἄλογοι lineæ, quòd ſi qua-

ſi quadrata quidem non fuerint, verum aliæ quæpiam ſuperficies ſiue figuræ rectilineæ, tunc vero lineæ illæ quæ deſcribunt quadrata æqualia figuris rectilineis, vocentur ἄλογοι.

Poſtulatum, ſiue petitio.

Poſtulatur quemlibet magnitudinē totius poſſe multiplicari, donec quamlibet magnitudinem eiuſdem generis excedat.

Axiomata, ſiue pronunciata.

1.

Magnitudo quotcunque magnitudines metiens, compoſitam quoque ex ipſis metitur.

2.

Magnitudo quamcunq; magnitudinem metiens, metitur quoque omnem magnitudinem, quam illa metitur.

3

Magnitudo metiens totam magnitudinem, & ablatam; metitur, & reliquam.

Problema 1. Propoſitio 1.

Duabus magnitudinibus inæqualibus propoſitis, ſi de maiore detrahatur plus dimidio; & rurſus de reſiduo iterum detrahatur plus dimidio; idque ſemper fiat: relinquetur tādem quędam magnitudo minor altera minore ex duabus propoſitis.



Theo-

Theorema 2. Propoſitio 2.

Duabus magnitudinibus propoſitis inęqualibus, ſi detrahatur ſemper minor de maiore, alterna quadam detractione; neque reſiduum vnquam metiatur id, quod ante ſe metiebatur; incommenſurabiles ſunt illæ magnitudines.

Problema 1. Propoſitio 3.

Duabus magnitudinibus commenſurabilibus datis, maximam ipſarum communem menſuram reperire.



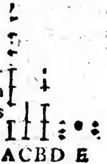
Problema 2. Propoſitio 4.

Tribus magnitudinibus commenſurabilibus datis, maximam ipſarum communem menſuram reperire.



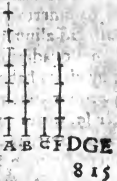
Theorema 3. Propoſitio 5.

Commenſurabiles magnitudines inter ſe proportionem eam habent, quam habet numerus ad numerum.



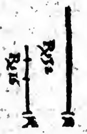
Theo-

Theorema 4. Propoſitio 6.



Si duæ magnitudines proportionem eam habent inter ſe, quam numerus ad numerum: commenſurabiles ſunt illæ magnitudines.

Theorema 5. Propoſitio 7.



Incommenſurabiles magnitudines inter ſe proportionem non habent, quam numerus ad numerum.

Theorema 6. Propoſitio 8.

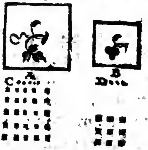
Si duæ magnitudines inter ſe proportionem non habent, quam numerus ad numerum: incommenſurabiles illæ ſunt magnitudines.

Theorema 7. Propoſitio 9.

Quadrata, quæ deſcribitur à rectis lineis lõgitudine commenſurabilibus: inter ſe pro-

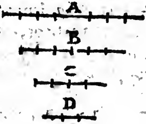
porti-

portionem habent, quam numerus quadratus ad alium numerum quadratum. Et quadrata habentia proportionem inter ſe, quam quadratus numerus ad numerum quadratum, habent quoque latera longitudine commenſurabilia. Quadrata verò quæ deſcribũtur à lineis longitudine incommenſurabilibus; proportionem non habent inter ſe, quã quadratus numerus ad numerũ alium quadratum. Et quadrata non habentia proportionem inter ſe, quam numerus quadratus ad numerum quadratum, neque latera habebunt longitudine commenſurabilia.



Theorema 8. Propoſitio 10.

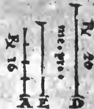
Si quatuor magnitudines fuerint proportionales; prima verò ſecundæ fuerit commẽſurabilis; tertia quoque quartæ commẽſurabilis erit; quòd ſi prima ſecũdæ fuerit incommenſurabilis; tertia quoque quartæ incommenſurabilis erit.



Theorema 3. Propoſitio 11.

Propoſitæ lineæ rectæ (quam ῥητὴν vocari dixi-

diximus) reperire duas lineas rectas incommenſurabiles, alteram quidẽ longitudine tantùm, alteram verò non longitudine tantũ, ſed etiam potentia incommẽſurabilem.



Theorema 9. Propoſitio 12.

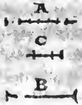


Magnitudines, quæ eidem magnitudini ſunt commẽſurabiles; inter ſe quoque ſunt commenſurabiles.

6 D.....4 F..
4 E.....8 G..
3H...
2K..
4L...

Theorema 10. Propoſitio 13.

Si ex duabus magnitudinibus hæc quidem commenſurabilis ſit tertiæ magnitudini, illa verò eidem incommenſurabilis, incommẽſurabiles ſunt illæ duæ magnitudines.



Theorema 11. Propoſitio 14.

Si duarum magnitudinum commenſurabilium, altera fuerit commenſurabilis magnitu-

nitudini alteri cupiam tertiæ; reliqua quoq; magnitudo eidem tertiæ incommenſurabilis erit.



Theorema 12. Propoſitio 15.

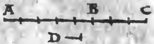
Si quatuor rectæ lineæ proportionales fuerint; poſsit autem prima pluſquam ſecunda tanto, quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine: tertia quoque potuerit pluſquam quarta tanto, quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine. Quòd ſi prima poſsit pluſquam ſecūda quadrato rectæ lineæ ſibi longitudine incommenſurabilis: tertia quoque poterit pluſquā quarta quadrato rectæ lineæ ſibi incommenſurabilis longitudine.



Theorema 13. Propoſitio 16.

Si duæ magnitudines commenſurabiles componantur; tota magnitudo compoſita ſingulis partibus commenſurabilis erit; Quòd ſi tota magnitudo compoſita alterutri parti commenſurabilis fuerit; illæ duæ quo-

quoque partes commẽſurabiles erunt.



Theorema 14. Propoſitio 17.

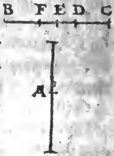
Si duæ magnitudines incommenſurabiles componantur, ipſa quoque tota magnitudo ſingulis partibus componentibus incõmenſurabilis erit. Quòd ſi tota alteri parti incõmenſurabilis fuerit illæ quoque primæ magnitudines inter ſe incommenſurabiles erunt.



Theorema 15. Propoſitio 18.

Si fuerint duæ rectæ lineæ inæquales, & quartæ parti quadrati, quod deſcribitur à minore, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quãtum eſt alterum latus ipſius parallelogrammi: ſi præterea parallelogrammum ſui applicatione diuidat lineam illam in partes inter ſe commenſurabiles longitudine, illa maior linea tanto plus poteſt quam minor, quantum eſt quadratum lineæ ſibi commẽſurabilis longitudine. Quòd ſi maior plus poſſit quam minor, tanto, quantum eſt quadratum lineæ ſibi commenſurabilis longitudine; & præterea quartæ parti quadrati li-

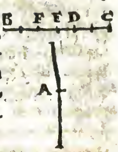
neæ minoris, æquale parallelogrammum applicetur ad maiorem, ex qua maiore tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius parallelogrammi: parallelogrammũ ſui applicatione diuidit maiorem in partes inter ſe longitudine commenſurabiles.



Theorema 16. Propoſitio 19.

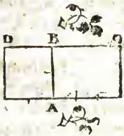
Si fuerint duæ rectæ lineæ inæquales; quartæ autem parti quadrati lineæ minoris, æquale parallelogrammorum ad lineam maiorem applicetur, ex qua linea tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus eiuſdem parallelogrammi: ſi parallelogrammum præterea ſui applicatione diuidat lineam in partes inter ſe longitudine incommenſurabiles, maior illa linea tantò plus poteſt quàm minor quantum eſt quadratum lineæ ſibi minori commenſurabiles longitudine. Quòd ſi maior linea tantò plus poſſit quàm minor, quantum eſt quadratum lineæ incommenſurabilis ſibi longitudine: & præterea quartæ parti quadrati lineæ minoris æquale parallelogrammum applicetur ad maiorem ex

ex qua tantum excurrat extra latus parallelogrammi, quantum eſt alterum latus ipſius: parallelogrammum ſui applicatione diuidit maiorem in partes inter ſe incõmenſurabiles longitudine.



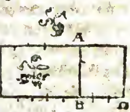
Theorema 17. Propoſitio 20.

Superficies rectanguli contenta ex lineis rectis rationalibus longitudine commenſurabilibus ſecundum vnum aliquem modum ex antedictis, rationalis eſt.



Theorema 18. Propoſitio 21.

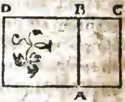
Si rationale ad lineam rationalem applicetur; habebit alterum latus lineam rationalem & commenſurabilem longitudine lineæ cui rationale parallelogrammum applicatur.



Theorema 19. Propoſitio 22.

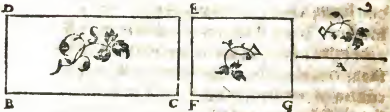
Superficies rectangula contenta duabus lineis rectis rationalibus potẽtia tantũ cõmen-

ſurabilibus, irrationalis eſt. Linea autem quæ illam ſuperficiem poteſt, irrationalis & ipſa eſt; vocetur verò media.



Theorema 20. Propoſitio 23.

Quadrati lineæ mediæ ad lineam rationalem applicâti, alterum latus eſt linea rationalis, & incommenſurabilis longitudine lineæ, ad quam applicatur.



Theorema 21. Propoſitio 24.

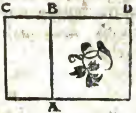
Linea recta mediæ commẽſurabilis, eſt ipſa quoque media.



Theo-

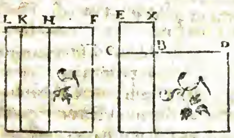
Theorema 22. Propo-
ſitio 25.

Parallelogrammum re-
ctangulum contentum
ſub rectis lineis medijs
longitudine commen-
ſurabilibus, medium eſt.



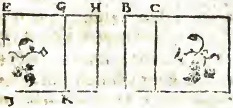
Theorema 23. Propoſitio 26.

Parallelogrammum rectangulum compre-
henſum
ſub dua
bus lineis
medijs
potentia
tantum
commen
ſurabilibus; NMG
vel rationale eſt, vel medium.



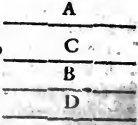
Theorema 24. Propoſitio 27.

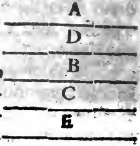
Medium
non eſt
maius,
quàm me
dium ſu-
perficie
rationali.



Problema 4. Propoſitio 28.

Medias lineas inuenire potentia tantùm commenſurabiles rationale comprehendentes.





Problema 5. Propoſitio 29.

Medias lineas inuenire potentia tantùm commenſurabiles, medium comprehendentes.

Problema 6. Propoſitio 30.

Reperire duas rationales potentia tantum commẽſurabiles huiuſmodi, vt maior ex illis poſſit plus, quàm minor, quadrato rectæ lineæ ſibi commenſurabilis longitudine.



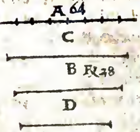
Problema 7. propoſitio 31.

Inuenire duas rationales potentia tantùm commenſurabiles; ita vt maior, quam minor plus poſſit, quadrato rectæ lineæ ſibi longitudine incommenſurabilis.

pro-

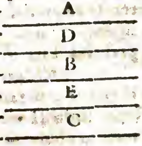
Problema 8. propoſitio 32.

Reperire duas lineas medias potentia tantùm commenſurabiles rationalem ſuperficiem continentes tales in quam, vt maior poſſit plus, quàm minor, quadrato rectæ lineæ ſibi commenſurabilis longitudine.



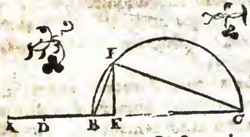
Problema 9. Propopoſitio 33.

Inuenire duas lineas medias potentia tantùm cómenſurabiles, quæ mediam ſuperficiem contineant, ita vt maior plus poſſit, quam minor, quadrato rectæ lineæ ſibi longitudine commenſurabilis.



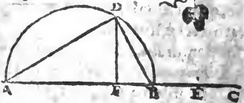
Problema 10. Propoſitio 34.

Inuenire duas rectas lineas potentia incommenſurabiles; quarum quadrata ſimul compoſita faciant ſuperficiem rationalem: Rectangulum vero ſub ipſis contentum, faciant medium.



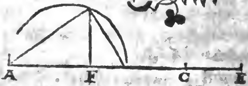
Problema 11. Propoſitio. 35.

Reperire lineas duas rectas potentia incommenſurabiles, conficientes compoſitum ex ipſarum quadratis medium parallelogrammũ verò ex ipſis contentum rationale.



Problema 12. Propoſitio 36.

Reperire duas lineas rectas potentia incommenſurabiles, conficientes id, quod ex ipſarum quadratis componitur, medium, parallelogrãmum ex ipſis contentum, mediũ; quod præterea parallelogrammum ſit incommẽſurabile compoſito ex quadratis ipſarum.



PRIN-

PRINCIPIVM SENARIORUM per compoſitionem, & Syntheſin.

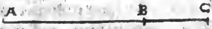
Theorema 25. Propoſitio 37.

Si duæ rationales potentia tantùm commẽſurabilis componantur; tota irrationalis erit. Vocetur autem Binomium, vel ex binis nominibus.



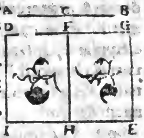
Theorema 26. Propoſitio 38.

Si duæ mediæ potentia tantùm commenſurabiles, rationale continentes, componantur; tota linea eſt irrationalis, vocetur autem ex binis medijs prima.



Theorema 27. Propoſitio 39.

Si duæ mediæ potentia tantùm commẽſurabiles medium continentes cõponantur; tota linea eſt irrationalis: vocetur autẽ ex binis medijs ſecunda.



Theorema 28. Propoſitio 40.

Si duæ rectæ lineę potentia incommenſurabiles componantur; conficientes compoſitum ex quadratis ipſarum, rationale paral-

lelogrammum verò ex ipſis contentum, medium: tota linea recta eſt irrationalis. Vocetur autem linea maior.



Theorema 29. Propoſitio 41.

Si duæ rectæ lineæ potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis, medium: id verò, quod fit ex ipſis, rationale; tota recta linea eſt irrationalis erit. Vocetur autem potens rationale & medium.



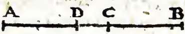
Theorema 30. Propoſitio 42.

Si duæ rectæ lineę potentia incommenſurabiles componantur, conficientes compoſitum ex ipſarum quadratis medium; & quod continetur ex ipſis, medium; & præterea incommenſurabile compoſito ex quadratis ipſarum: tota recta linea eſt irrationalis. Vocetur autem bina media potens.



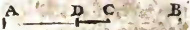
Theorema 31. Propoſitio 43.

Quæ linea ex binis nominibus vocata, in vnico tantum puncto diuiditur in ſua nomina.



Theorema 32. Propoſitio 44.

Quæ ex binis medijs prima, in vnica tantũ puncto diuiditur in ſua nomina.



Theorema 33. Propoſitio 45.



Quæ ex binis medijs ſecunda, in vnico tantum puncto diuiditur in ſua nomina.

Theorema 34. Propoſitio 46.

Linea maior in vnico tantùm puncto diuiditur in ſua nomina.

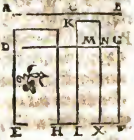


Problema 35. Propoſitio 47.

Linea potens rationale & medium, in vnico tantùm puncto, diuiditur in ſua nomina.



Theorema 36. Propoſitio 48.



Linea potens duo media in vnico tantum puncto diuiditur in ſua nomina.

DE-

DEFINITIONES ſecundæ, nempe binorum nominum.

Propoſita linea rationali; & linea ex binis nominibus vocata, diuiſa, in ſua nomina, cuius maius nomen, id eſt, maior portio, poſſit pluſquam minus nomen; quadrato lineæ ſibi, maiori inquam nomini, commenſurabilis longitudine.

1.

Si quidem maius nomen fuerit commenſurabile longitudine propoſitæ lineæ rationali; Vocetur tota linea compoſita ex binis nominibus, prima.

2.

Si verò minus nomen, id eſt, minor portio, fuerit commenſurabile longitudine propoſitæ lineæ rationali; Vocetur tota linea ex binis nominibus ſecunda.

3.

Si verò neutrum ipſorum nominum fuerit commenſurabile longitudine propoſitæ lineæ rationali; Vocetur tota ex binis nominibus tertia.

Rurſus ſi maius nomen poſſit pluſquam minus nomen, quadrato lineæ ſibi incommenſurabiles longitudine.

4.

Si quidem maius nomen ſit commenſurarabi-

rabile longitudine propoſitæ lineæ rationali; Vocetur tota linea ex binis nominibus quarta.

5.

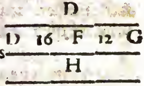
Si verò minus nomen fuerit commenſurabile longitudine lineæ rationali; Vocetur tota ex binis nominibus quinta.

6.

Si verò neutrum ipſorum nominum fuerit longitudine commenſurabile: propoſitæ lineæ rationali; Vocetur tota ex binis nominibus ſexta.

Problema 13. Propoſitio 49.

Reperire lineam ex binis nominibus primam.

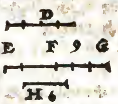


12 4
A.....C.....B
16

Problema 14. Propoſitio. 50.

9 3
A......C...B
12

Reperire ex binis nominibus ſecundam.

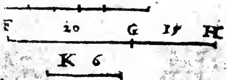


Pro-

Problema 15. Propoſitio 51.

15 5
A..........C.....
20
D
....
E

reperire ex binis nomi n b' tertiã



Problema 16. Propoſitio 52.

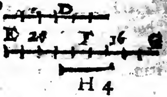
10 6
A..........C.......B
16

Reperire ex binis nominibus quartam.



16 4
Problema 17. Propoſitio 53.
A..............C.....
20

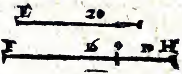
Reperire ex binis nominibus quintam.



10 6
A..........C......B
Problema 18. Propoſitio 54.
16
D...............
20

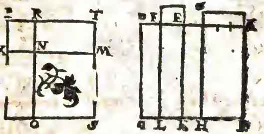
Repe-

Reperire ex binis nominibus ſextam.



Theorema 37. Propoſitio 55.

Si ſuperficies contenta fuerit ſub rationali, & ex binis nominibus prima recta linea, quæ illam ſuperficiem poteſt, eſt irrationalis, quæ ex binis nominibus vocatur.



Theorema 38. Propoſitio 56.

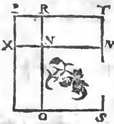
Si ſuperficies cõtẽta fuerit ſub linea rationali, & ex binis nominibus ſecunda; Recta linea potens illã ſuperficiem, eſt irrationalis, quæ ex binis medijs prima vocatur.

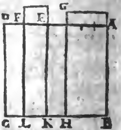


Theo-

Theorema 39. Propoſitio 57.

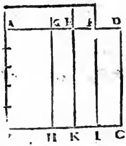
Si ſuperficies contineatur ſub rationali, & ex binis nominibus tertia; recta linea, qua illam ſuperficiem poteſt, eſt irrationalis; quæ ex binis medijs dicitur tertia.

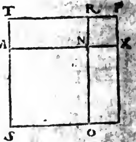




Problema 40. Propoſitio 58.

Si ſuperficies contineatur ſub rationali, & ex binis nominibus quarta; recta linea potens, ſuperficiem illam, eſt irrationalis; quæ dicitur maior.

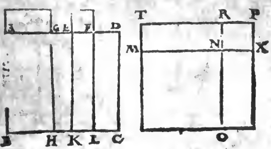




Theorema 41. Propoſitio 59.

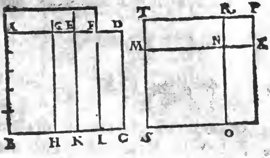
Si ſuperficies contineatur ſub rationali, & ex binis

binis nominibus quarta; recta linea, quæ illam ſuperficiem poteſt, eſt irrationalis; quæ dicitur potens rationale, & medium.



Theorema 42. Propoſitio 60.

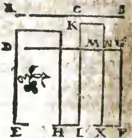
Si ſuperficies contineatur ſub rationali, & ex binis nominibus ſexta; Recta linea, quæ illam ſuperficiem poteſt, eſt irrationalis; quæ dicitur potens, bina media.



Theorema 43. Propoſitio 61.

Quadratum eius lineæ, quæ eſt ex binis nomini-

minibus, ad lineam rationalẽ applicatum, facit latitudinem ex binis nominibus primam.



Theorema 44. Propoſitio 62.

Quadratum, eius quæ eſt ex binis medijs prima, ad lineam rationalem lineam applicatum, facit latitudinem ex binis nominibus ſecundam.



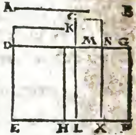
Theorema 45. Propoſitio 63.

Quadratum eius, quæ eſt ex binis medijs ſecunda, ad lineam rationalem applicatum, facit latitudinẽ ex binis nominibus tertiam.



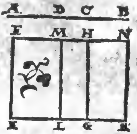
Theorema 46. Propoſitio 64.

Quadratum lineæ maioris ſecundum lineam rationalem applicatum, facit latitudinem ex binis nominibus quartam.



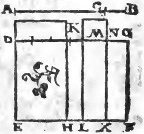
Theo-

Theorema 47. Propoſitio 65.



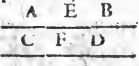
Quadratum lineæ potẽtis rationale, & medium ſecundum rationalem applicatum, facit latitudinem ex binis nominibus quintam.

Theorema 48. Propoſitio 66.



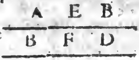
Quadratum lineæ potentis duo media, ſecundum rationalem applicatum, facit latitudinem ex binis nominibus ſextam.

Theorema 49. Propoſitio 67.



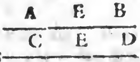
Linea longitudine commenſurabilis, ei lineę quę eſt ex binis nominbus; & ipſa ex binis nominibus eſt, atque in ordine eadem.

Theorema 50. Propoſitio 68.



Linea longitudine commenſurabilis alteri lineæ quæ eſt ex binis Medijs; & ipſa ex binis medijs eſt, atque in ordine eadem.

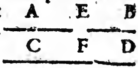
Theorema 51. Propoſitio 69.



Linea commenſurabilis lineæ maiori, & ipſa maior eſt. K 2 Theo-

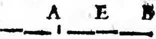
Theorema 52. Propoſitio 70.

Linea commenſurabilis lineæ potenti rationale & medium, eſt & ipſa linea potens rationale & medium.



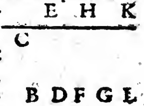
Theorema 53. Propoſitio 71.

Linea commenſurabilis lineæ potenti duo media, eſt & ipſa linea potens duo media.



Theorema 54. Propoſitio 72.

Si duæ ſuperficies, rationalis, & media ſimul componantur, linea quę totam ſuperficiem compoſitam poteſt, eſt vna ex quatuor irrationalibus; vel ea, quæ dicitur ex binis nominibus, vel ea, quæ ex binis medijs prima, vel linea maior, vel linea potens, rationale & medium.

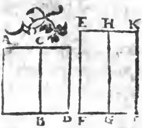


Theorema 55. Propoſitio 73.

Si duæ ſuperficies mediæ inter ſe incommen-

ſura-

ſurabiles ſimul componantur; fiunt reliquę duę lineæ irrationales; & ex binis medijs ſecunda, vt bina media potens.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Chriſt. Clauio.

Ex his omnibus facilè colligitur, quod linea ea, quæ eſt ex binis nominibus, & cæteræ ipſam ſubſequentes, lineæ irrationales, neque ſunt eadem cum linea media, neque ipſæ inter ſe ſunt eadem.

Nam quadratum lineæ mediæ, ad lineam rationalem comparatum & applicatum, efficit alterum latus, lineam rationalem, ſeu latitudinem rationalem, ipſi lineæ rationali (hoc eſt, lineæ, ad quã applicatur) longitudine incommenſurabilem: per propoſ. 23. libri decimi.

Quadratum verò eius lineæ, quæ eſt ex binis nominibus, ad rationalem, applicatum, efficit alterum latus, & lineam, ſeu latitudinem ex binis nominibus primam: per 61.

Quadratum verò eius, quæ eſt ex binis medijs prima ad Rationalem applicatum, latitudinem efficit ex binis nominibus ſecundam: per 62.

Quadratum verò eius, quæ eſt ex binis medijs ſecunda, ad rationalem applicatum, latitudinem efficit

ſicit ex binis nominibus tertiam: per 63.

Quadratum lineæ maioris, ad rationalem applicatum, latitudinem efficit ex binis nominibus quartam: per 64.

Quadratum vero eius, quæ rationale, & medium poteſt, ad rationalem applicatum; efficit alterum latus, ſeu latitudinem ex binis nominibus quintam: per 65.

Quadratum denique lineæ eius, quæ bina media poteſt, ad rationalem applicatum, efficit alterum latus, ſeu latitudinem ex binis nominibus ſextam: per 66.

Cùm igitur hæ latitudines (quæ à nonnullis latera dicuntur.) differant, & à latitudine media & inter ſe, à latitudine quidem media, quod hæc rationalis ſit, illæ verò irrationales, inter ſe autem, quod in ordine non ſint eædem cum iis ex binis nominibus: manifeſtum eſt omnes ipſas irrationales lineas, de quibus hactenus dictum eſt, inter ſe differentes eſſe.

PRINCIPIVM SENARIORVM per detractionem, & aphæreſin.

Theorema 56. Propoſitio 74.

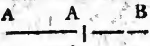
Si de linea rationali detrahatur rationalis, potentia tantùm commenſurabilis ipſi toti; Reſidua eſt irrationalis: Vocetur autem Reſiduum, hoc eſt, apotome.



Theo-

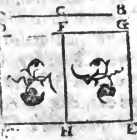
Theorema 57. Propoſitio 75.

Si de linea media detrahatur media, potentia tantum commenſurabilis toti lineæ; quæ verò detracta eſt, cúm tota contineat ſuperficiem rationalem; Reſidua eſt irrationalis. Vocetur autem Reſiduum mediũ primum: hoc eſt, mediæ apotome prima.



Theorema 58. Propoſitio 76.

Si de linea media detrahatur media, potentia tantùm commenſurabilis tóti; quæ verò detracta eſt, cum tota có tineat ſuperficiem medium: Reliqua eſt irrati- tionalis. Vocetur autem Reſiduum medium ſecundúm, hoc eſt, mediæ apotome ſecunda.



Theorema 59. Propoſitio 77.

Si de linea recta detrahatur recta, potentia incommenſurabilis toti; compoſitum autem ex quadratis totius lineæ, & lineæ detractæ, ſit rationale; parallelogrammum verò ex

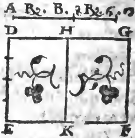
rò ex eiſdem contentum, ſit medium: Reliqua linea erit irrationalis. A C B Vocetur autem linea minor.

Theorema 60. Propoſitio. 78.

Si de linea recta detrahatur recta, potentia incõmenſurabilis toti lineæ; compoſitum autem ex quadratis totius, & lineæ detractæ ſit medium; parallelogrammum verò bis ex eiſdem contentum, ſit rationale: Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie rationali totam ſuperciem mediam. A C B

Theorema 61. Propoſitio 79.

Si de linea recta detrahatur recta potentia incommenſurabilis toti lineæ; compoſitũ autem ex quadratis totius, & lineæ detractæ, ſit medium: Parallelogrammum verò bis ex iiſdem ſit etiam medium: præterea ſint quadrata ipſarum incommenſurabilia parallelogrammo bis ex iiſdem contento, Reliqua linea eſt irrationalis. Vocetur autem linea faciens cum ſuperficie media totam ſuperficiem mediam.



Theo-

Theorema 62. Propoſitio 80.

Reſiduo vnica tantùm linea recta coniungitur rationalis, potentia tantùm commenſurabilis toti lineæ.

A B B D

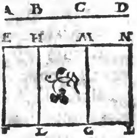
Theorema 63. Propoſitio 81.

Reſiduo medio primo vnica tantùm linea coniũgitur media, potentia tantùm commenſurabilis toti, ipſa cum tota rationale continens.

A B C D

Theorema 64. Propoſitio 82.

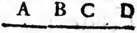
Reſiduo medio ſecundo vnica tantùm coniũgitur recta linea media, potẽtia tantùm commenſurabilis toti, ipſa cum tota medium continens.



Theorema 65. Propoſitio 83.

Lineæ minori vnica tantum recta linea coniungitur, potentia incommenſurabilis toti, faciens cum tota compoſitum ex quadratis ipſa-

ipſarum rationale: id verò parallelogrammum, quod bis ex ipſis fit, medium.



Theorema 66. Propoſitio 84.

Lineæ facienti cum ſuperficie rationali totam ſuperficiem mediam, vnica tantùm cõiungitur linea recta, potentia incommenſurabilis toti; faciens autem cum tota compoſitum ex quadratis ipſarum, medium; Id verò, quod fit bis ex ipſis, rationale.



Theorema 67. Propoſitio 85.

Lineæ cum media ſuperficie facienti totam ſuperficiem mediã, vnica tantum coniungitur linea, potentia toti incõmenſurabilis, faciens cũ tota compoſitum ex quadratis ipſarum, medium, id verò, quod bis ex ipſis etiam medium: & præterea faciens compoſitum ex quadratis ipſarum incommenſurabile ei, quod fit bis ex ipſis



DE-

DEFINITIONES.

TERTIAE, NEMPE APOTOMARUM, *ſeu Reſiduorum.*

Propoſita linea rationali, & Reſiduo, ſi tota nēpe compoſita ex ipſo Reſiduo, & linea illi coniuncta, ſeu congruente, plus poſſit, quā coniuncta, quadrato rectæ lineæ ſibi longitudine commenſurabilis;

I.

Si quidē tota lineæ propoſitæ rationali ſit longitudine commenſurabilis; Vocetur reſiduum primum, ſeu Apotome prima.

2.

Si verò coniuncta fuerit longitudine commenſurabilis propoſitæ rationali; ipſa autem tota plus poſſit, quam coniuncta, quadrato lineæ, ſibi longitudine commenſurabilis: Vocetur Reſiduum ſecundū, ſeu Apotome ſecunda.

3

Si verò neutra linearum fuerit longitudine commenſurabilis propoſitæ rationali; poſſit autem ipſa tota pluſquam coniuncta, quadrato lineæ ſibi longitudine commenſurabilis; Vocetur Reſiduum tertium, ſeu Apotome tertia.

Rurſus ſi tota poſſit plus, quàm coniuncta ſeu congruens, quadrato rectæ lineæ ſibi longitudine incommenſurabilis;

4 Et

4.

Et quidem ſi tota fuerit longitudine commenſurabilis ipſi rationali; Vocetur Reſiduum quartum, ſeu Apotome quarta.

5.
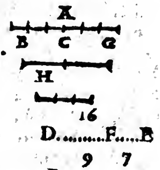
Si verò coniuncta fuerit longitudine cōmenſurabilis ipſi rationali; & tota plus poſſit, quam coniuncta, quadrato lineæ ſibi lógitudine incommenſurabilis, Vocetur Reſiduum quintum, ſeu Apotome quinta.

6.

Si denique neutra linearum fuerit commenſurabilis longitudine ipſi rationali; fueritque tota potentior, quam coniuncta; quadrato lineæ ſibi longitudine incommenſurabilis; Vocetur Reſiduum ſextum, ſeu Apotome ſexta.

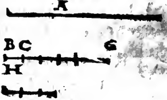
Problema 19. Propoſitio 86.

Reperire primum Reſiduum.



Problema 20. Propoſitio 87.

Reperire ſecundum Reſiduum.



Pro

D.........F.....G

27 9

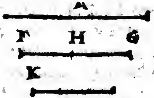
E.........

12

B.........F.....C

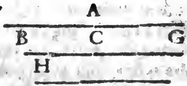
9 7

Problema 21. Propoſitio 88.



Reperire tertium Reſiduum.

Problema 22. Propoſitio 89.

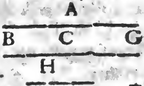


D.........F.....E

16 4

Reperire quartum Reſiduum.

Problema 23. Propoſitio 90.

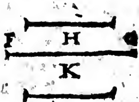


D.........F.....E

25 7

Reperire quintum Reſiduum.

Problema 24. Propoſitio 91.



Reperire ſextum Reſiduum.

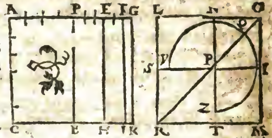
E.......... 13

B......D.....C

Theo-

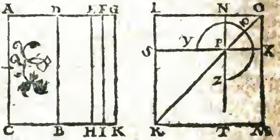
Theorema 68. Propoſitio 92.

Si ſuperficies contineatur ſub linea rationali, & reſiduo primo; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum.



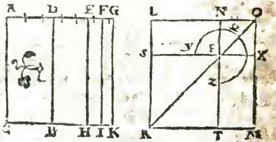
Theorema 69. Propoſitio 93.

Si ſuperficies contineatur ſub linea rationali, & reſiduo ſecundo; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum mediale primum, ſeu mediæ Apotome prima.



Theorema 70. Propoſitio 94.

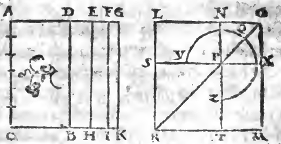
Si ſuperficies cōtineatur ſub linea rationali,



tionali, & reſiduo tertio; recta linea, quæ illam ſuperficiem poteſt, eſt reſiduum mediale ſecundum, ſeu mediæ eſt Apotome ſecunda.

Theorema 71. Propoſitio 95.

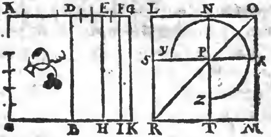
Si ſuperficies contineatur ſub linea rationali, & reſiduo quarto; recta linea, quæ



illam ſuperficiem poteſt, eſt linea minor.

Theorema 72. Propoſitio 96.

Si ſuperficies contineatur ſub linea rationali, & reſiduo quinto; recta linea, quæ illam ſuperficiem poteſt, eſt ea, quæ dicitur cum

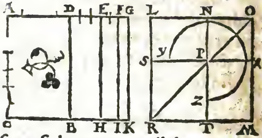


rationali ſuperficie faciẽs totam medialem

Theorema 73. Propoſitio 97.

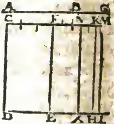
Si ſuperficies cõtineatur ſub linea rationali, &

& Reſiduo ſexto; recta linea, quæ illam ſuperficiē poteſt, eſt ea, quæ dicitur faciens cum



mediali ſuperficie totam medialem.

Theorema 74. Propoſitio 98.



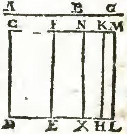
Quadratum reſidui ad lineam rationalem applicatum, facit alterum latus reſiduum primum.

Theorema 75. Propoſitio 99.



Quadratum reſidui medialis primi ad rationalem applicatum, facit alterum latus, reſiduum ſecundum.

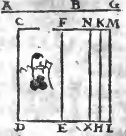
Theorema 76. Propoſitio 100.



Quadratum reſidui medialis ſecundi ad rationalem applicatum, facit alterum latus reſiduum tertium.

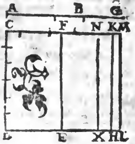
Theo.

Theorema 77. Propoſitio 101.



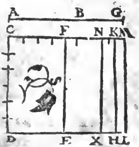
Quadratum lineę minoris ad rationalem applicatum, facit alterum latus reſiduum quartum.

Theorema 78. Propoſitio 102.



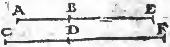
Quadratum lineæ cum rationali ſuperficie facientis totam medialem, ad rationalem applicatum, facit alterum latus reſiduum quintum.

Theorema 79. Propoſitio 103.



Quadratum lineæ cum mediali ſuperficie faciẽtis totam medialem, ad rationalem applicatum, facit alterum latus reſiduum ſextum.

Theorema 80. Propoſitio 104.

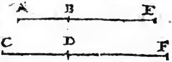


Recta linea reſiduo commenſurabilis longitudine; eſt & ipſa reſiduum, ſeu in ordine eadem.

Theorema 81. Propoſitio 105.

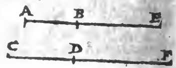
Recta linea commenſurabilis reſiduo me-

diali, eſt & ipſa reſiduum mediale, & eiuſdem ordinis; ſeu in ordine eadem.



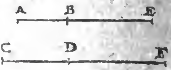
Theorema 82. Propoſitio 106.

Recta linea cõmenſurabilis lineæ minori: eſt & ipſa linea minor.



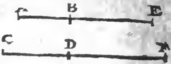
Theorema 83. Propoſitio 107.

Recta linea commenſurabilis lineæ cum rationali ſuperficie facienti totam medialem; eſt & ipſa linea cum rationali ſuperficie faciens totam medialem.



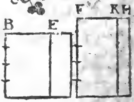
Theorema 84. Propoſitio 108.

Recta linea commenſurabilis lineæ cum mediali ſuperficie facienti totam medialem; eſt & ipſa cum mediali ſuperficie faciens totam medialem.



Theorema 85. Propoſitio 109.

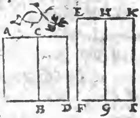
Si de ſuperficie rationali detrahatur ſuperficies medialis; recta linea, quæ reliquam ſuperficiem poteſt, eſt alterutra ex duabus irrationalibus, aut reſiduum, aut linea minor.



Theo-

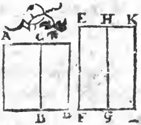
Theorema 86. Propoſitio 110.

Si de ſuperficie mediali detrahatur ſuperficies rationalis; aliæ duæ irrationales fiunt, aut reſiduum mediale primum, aut cum rationali ſuperficies totam medialem.



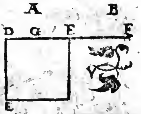
Theorema 87. Propoſitio 111.

Si de ſuperficie mediali detrahatur ſuperficies medialis, quæ ſit incommēſurabilis toti; reliquæ duæ fiunt irrationales, aut reſiduum mediale ſecũdum, aut cum mediali ſuperficie faciēs totam medialem.



Theorema 88. Propoſitio 112.

Linea, quæ reſiduum dicitur, non eſt eadem cum ea, quæ dicitur Binomium.



SCHOLIVM EX THEONE, ZAMberto, Campano, & P. Clauio.

Ex his demonſtratis facilè intelligitur, quod recta linea, quæ reſiduum dicitur, & cæteræ quinq; eam conſequẽtes irrationales, neq; lineæ mediali, neq; ſibi ipſæ inter ſe ſunt eædem.

Nam quadratum lineæ medialis ſecundum rationalem applicatum, facit alterum latus, rationalem lineam, longitudine incommenſurabilem ei, ſeu ad quam applicatur, per propoſ. 23. libri decimi.

Quadratum, verò reſidui ſecundum rationalem applicatum, facit alterum latus, reſiduum primum per 98.

Quadratum verò reſidui medialis primi ſecundum rationalem applicatum, facit alterum latus, reſiduum ſecundum per 99.

Quadratum verò reſidui medialis ſecundi, facit alterum latus, reſiduum tertium, per 100.

Quadratum verò lineæ minoris, facit alterum latus reſiduum quartum, per 101.

Quadratum verò lineæ cum rationali ſuperficie facientis totam medialem facit alterum latus reſiduum quintum, per 102.

Quadratum verò lineæ cum mediali ſuperficie facientis totam medialem, ſecundum rationalem applicatum, facit alterum latus reſiduum ſextum, per 103.

Cùm

Cùm igitur dicta latera, quæ ſunt latitudines cuiuſq; parallelogrammi vnicuiq; quadrato æqualis, & ad rationalem, applicati, differant & à primo latere, & ipſa inter ſe, (nam à primo differunt: quoniam ſunt reſidua non eiuſdem ordinis) conſtat ipſas quoq; lineas irrationales inter ſe differentes eſſe. Et quoniam demonſtratum eſt, reſiduum non eſſe idem quod Binomium: quadrata autem reſidui, & quinq; linearum irrationalium illud conſequentium, ad rationalem applicata, faciunt altera latera ex reſiduis eiuſdem ordinis, cuius ſunt & reſidua, quorum quadrata applicantur rationali, ſimiliter & quadrata Binomij, & quinq; linearum irrationalium illud conſequentium, ad rationalem applicata, faciunt altera latera ex Binomijs eiuſdem ordinis, cuius ſunt & Binomia, quorum quadrata applicantur rationali. Ergo lineæ irrationales, quæ conſequuntur Binomium, & quæ conſequuntur reſiduum, ſunt inter ſe differentes. Quare dictæ lineæ omnes irrationales ſunt numero, 13. ſubſequentes.

1. *Media linea; quæ vulgò medialis appellatur: propoſ. 22.*
2. *Linea ex binis nominibus (vulgò Binomium;) cuius ſex ſunt ſpecies inuentæ: propoſ. 37.*
3. *Ex binis medijs prima; vulgò Bimediale primum; propoſ. 38.*
4. *Ex binis medijs ſecunda; vulgò Bimediale ſecundum: propoſ. 39.*
5. *Maior. propoſ. 40.*

6 Linea rationale, ac medium potens: propoſ. 41.

7. Bina media potens: propoſ. 42

8. Apotome (vulgò reſiduum:) cuius etiam ſpecies ſex ſunt reperta: propoſ 74

9. Mediæ Apotome prima; vulgò reſiduum: propoſ. 75

10. Mediæ Apotome ſecunda, vulgò reſiduum mediale ſecundum: propoſ 76

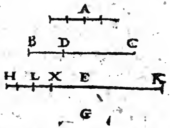
11. Minor: propoſ 77

12. Linea cum rationali medium totum efficiens; vulgò linea cum rationali ſuperficie totam medialem faciens: propoſ. 78

13. Cum medio medium totum efficiens; vulgò linea cum mediali ſuperficie totam medialem faciens: propoſ. 79

Theorema 89. Propoſitio 113.

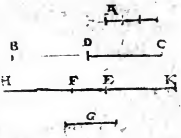
Quadratum lineæ rationalis ad Binomium applicatum, facit alterum latus reſiduum, cuius nomina ſunt commenſurabilia Binomij nominibus, & in eadem proportione præterea id, quod fit reſiduum, eundem ordinem retinet, quem Binomium.



Theorema 90. Propoſitio 114.

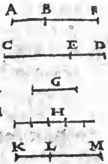
Quadratum lineæ rationalis ad reſiduum applicatum, facit alterum latus Binomium, cuius

cuius nomina ſunt commenſurabilia nominibus reſidui & in eadem proportione; præterea id, quod fit Binomium eſt eiuſdem ordinis, cuius & reſiduum.



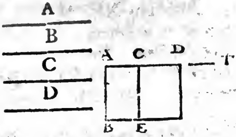
Theorema 91. Propoſitio 115.

Si parallelogrammum contineatur ex reſiduo, & Binomio, cuius nomina ſunt commenſurabilia nominibus reſidui, & in eadem proportione; recta linea, quæ illam ſuperficiem poteſt, eſt rationalis.



Theorema 92. Propoſitio. 116.

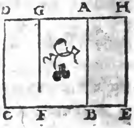
Ex linea media naſcuntur lineæ irrationales inumerabiles, quarum nulla vlli antedictarum eadem ſit.



Theorema 103. Propoſitio 117.

E..H..E
E...

Propoſitum ſit nobis demonſtrare, in figuris quadratis diametrum eſſe lõgitudine incommenſurabilem ipſi lateri.



FINIS ELEMENTI X.

EVCLI-

EVCLIDIS ELEMENTVM VNDECIMVM ET SOLIDORVM primum.

DEFINITIONES.

1.

Solidum, eſt quod longitudinem, latitudinem,& craſſitudinem habet.

2.

Solidi autem extremum eſt ſuperficies.

3.

Linea recta eſt ad planum recta, cùm ad rectas omnes lineas,à quibus illa tãgitur,quæque in propoſito ſunt plano, rectos angulos efficit.

4.

Planũ ad planũ rectum eſt,cùm rectæ lineæ, quæ cõmuni planorũ ſectioni ad rectos angulos in vno planorũ ducũtur, alteri plano ad recto ſunt angulos.

5.

Rectæ lineæ ad planum inclinatio eſt angulus acutus, ipſa inſiſtente linea, & adiuncta altera comprehenſus, cùm à ſublimi rectæ illius lineæ termino deducta fuerit perpendicularis,atque à puncto,quod perpendicularis in ipſo plano fecerit, ad propoſitæ illius lineæ extremum, quod in eodem eſt plano,altera recta linea fuerit adiuncta.

6.

Plani ad planum inclinatio, eſt angulus acutus rectis lineis contentus, quæ in vtroque planorum ad idem communis ſectionis pũctum ductæ, rectos ipſi ſectioni angulos efficiunt.

7.

Planum ſimiliter inclinatum eſſe ad planum, atq; alterum ad alterum dicitur, cùm dicti inclinationum anguli inter ſe ſunt æquales.

8.

Parallela plana ſunt, quæ inter ſe non incidunt, nec concurrunt.

9.

Similes figuræ ſolidæ ſunt, quæ ſimilibus planis, multitudine æqualibus continentur.

10.

Aequales, & ſimiles figuræ ſolidæ ſunt, quæ ſimilibus planis, multitudine & magnitudine æqualibus continentur.

11.

Solidus angulus eſt plurimum, quàm duarum linearum, quæ ſe mutuò contingant, nec in eadem ſint ſuperficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus eſt, qui pluribus, quam duobus planis angulis, in eodem plano non conſiſtentibus, ſed ad vnum punctum collectis continetur.

12. Pyra-

12.

Pyramis eſt figura ſolida, quæ planis continetur, ab vno plano ad vnum punctum collecta.

13.

Priſma figura eſt ſolida, quæ planis continetur, quorum aduerſa duo ſunt, & æqualia, & ſimilia, & parallela; alia verò parallelogramma.

14.

Sphæra eſt figura, quæ conuerſo circum quieſcentem diametrum ſemicirculo continetur, cùm in eundem rurſus locum reſtitutus fuerit, vnde moueri cœperat.

Aliter ex Theodoſio.

Sphæra eſt figura ſolida, ſub vna ſuperficie comprehenſa, ad quam ab vno puncto eorum, quæ intra figuram ſunt poſita, cadentes omnes rectæ lineæ, inter ſe ſunt æquales.

15.

Axis autem ſphæræ eſt, quieſcens illa linea recta, per centrum ducta, circum quam ſemicirculus conuertitur.

16

Centrum verò Sphæræ eſt idem, quod & ſemicirculi.

17.

Diameter autem ſphæræ eſt, recta quædam linea per centrum ducta, & vtrinque à ſphæræ ſuperficie terminata.

18. Conus

18.

Conus eſt figura, quæ ſub conuerſo circum quieſcens alterum latus eorum, quę rectum angulum continent, orthogonio triangulo continetur, cum in eundem rurſus locum illud triangulum reſtitutum fuerit, vnde moueri cœperat.

Atque ſi quieſcens recta linea æqualis ſit alteri, quæ circum rectum angulum conuertitur, orthogonius erit Conus: ſin minor, amblygonius: ſi verò maior, oxygonius.

19.

Axis autem Coni eſt, quieſcens illa recta linea, circum quam triangulum vertitur.

20.

Baſis verò Coni eſt, circulus qui à circumducta linea recta deſcribitur.

21.

Cylindrus eſt figura, quæ ſub conuerſo circumquieſcens alterum latus eorum, quæ rectum angulum continent, parallelogrammo orthogonio compræhenditur, cum in eundem ruſus locum reſtitutum fuerit illud parallelogrammum, vnde moueri cœperat.

22.

Axis autem Cylindri eſt quieſcens illa recta linea, circum quam parallelogrammum vertitur.

23. Baſes

23.

Baſes verò cylindri ſunt circuli, à duobus aduerſus lateribus, quæ circum aguntur, deſcripti.

24.

Similes coli, & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

25.

Cubus hexandrum eſt figura ſolida, quæ ſub ſex quadratis æqualibus continetur.

26.

Tetraedrum eſt figura ſolida, quæ ſub triangulis quatuor æqualibus, & æquilateris continetur.

27.

Octaedrum figura eſt ſolida, quæ ſub octo triangulis æqualibus, & æquilateris continetur.

28.

Dedecaedrum figura eſt ſolida, quæ ſub duodecim pentagonis æqualibus, æquilateris, & æquiangulis continetur.

29.

Eicoſaedrum figura eſt ſolida, quæ ſub triangulis viginti æqualibus, & ęquilateris cõtinetur.

30.

Parallelepipedum eſt figura ſolida, quæ ſub ſex figuris quadrilateris, quarum quæ ex adverſo, parallelæ ſunt, continetur.

31. Soli

31.

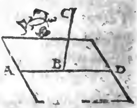
Solida figura in ſolida dicitur inſcribi, quando omnes anguli figuræ inſcriptæ conſtituuntur, vel in angulis, vel in lateribus, vel deniq; in planis figuræ, cui inſcribitur.

32.

Solida figura ſolidæ figuræ viciſſim circumſcribi dicitur, quando vel anguli, vel latera, vel deniq; plana figuræ circumſcriptæ tangunt omnes angulos figuræ, circum quam deſcribitur.

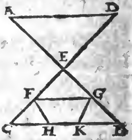
Theorema 1. Propoſitio 1.

Quædã rectæ lineæ pars in ſubiecto quidem non eſt plano, quædam verò in ſublimi.



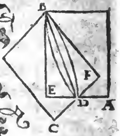
Theorema 2. Propoſitio 2.

Si duæ rectæ lineæ ſe mutuo ſecent, in vno ſunt plano: atque triangulum omne in vno eſt plano.



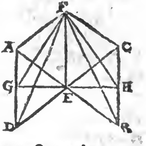
Theorema 3. Propoſitio 3.

Si duo plana ſe mutuò ſecent, communis eorum ſectio eſt recta linea.



Theo-

Theorema 4. Propoſitio 4.



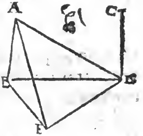
Si recta linea, rectis duabus lineis ſe mutuò ſecantibus, in communi ſectione ad rectos angulos inſiſtat: illa, ducto etiam per ipſas plano, ad angulos rectos erit.

Theorema 5. Propoſitio 5.



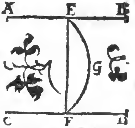
Si recta linea, rectis tribus lineis ſe mutuò tangentibus, incommuni ſectione ad rectos angulos inſiſtat: illæ tres rectæ in vno ſunt plano.

Theorema 6. Propoſitio 6.



Si duæ rectę lineę eidem plano ad rectos ſint angulos: parallelæ erunt illæ rectæ lineæ.

Theorema 7. Propoſitio 7.



Si duæ ſint parallelæ rectæ lineæ, in quarum vtraq; ſumpta ſint quælibet punctà: illa linea, quę adhæc puncta adiungitur, in eodem eſt cum parallelis plano.

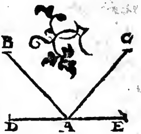
Theo-

Problema 2. Propoſitio. 12.



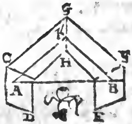
Dato plano, à puncto, quod in illo datum eſt, ad rectos angulos rectam lineam excitare.

Theorema 11. Propoſitio 13.



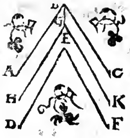
Dato plano, à pūcto quod in illo datum eſt, duæ rectæ lineæ ad rectos angulos non excitabuntur, ad eaſdem partes.

Theorema 12. Propoſitio 14.



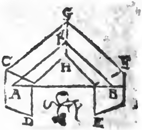
Ad quæ plana, eadem recta linea recta eſt; illa ſunt parallela.

Theorema 13. Propoſitio 15.



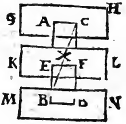
Si duæ rectæ lineæ ſe mutuo tangentes, ad duas rectas ſe mutuò tangentes ſint parallelæ, non in eodem conſiſtentes plano: parrallela ſunt, quæ per illas ducuntur, plana.

Theorema 14. Propoſitio 16.



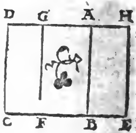
Si duo plana parallela plano quopiam ſecentur; communes illorum ſecti ones ſunt parallelæ.

Theorema 15. Propoſitio 17.



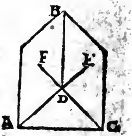
Si duæ rectæ lineæ parallelis planis ſecentur; in eaſdem proportiones ſecabuntur.

Theorema 16 Propoſitio 18.



Si recta linea plano cuipiam ad rectos ſit angulos; illa etiam omnia, quæ per ipſam plana, ad rectos eidem plano angulos erunt.

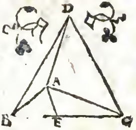
Theorema 17. Propoſitio 19.



Si duo plana ſe mutuò ſecantia, plano cuidam ad rectos ſint angulos; communis etiam illorum ſectio ad rectos eidem plano angulos erit.

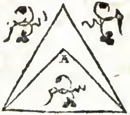
Theo-

Theorema 18. Propoſitio 20.



Si angulus ſolidus ſub planis tribus angulis contineatur: ex his duo quilibet, vtut aſſumpti, tertio ſunt maiores.

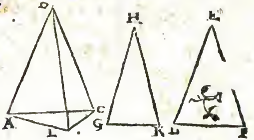
Theorema 19. Propoſitio 21.



Solidus omnis angulus ſub minoribus quàm rectis quatuor angulis planis, continetur.

Theorema 20. Propoſitio 22.

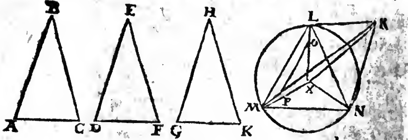
Si plani tres anguli æqualibus rectis contineantur lineis, quorum duo vt libet aſſumpti, tertio ſint maiores; triangulum conſtitui poteſt ex lineis æquales illas rectas coniungentibus.

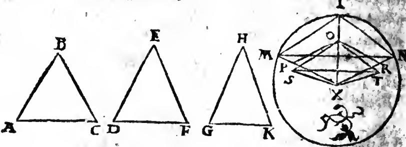


Problema 3. Propoſitio 23.

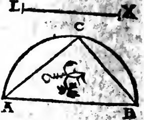
Ex planis tribus angulis, quorum duo, vt libet aſſumpti, tertio ſint maiores, ſolidum

angulum conſtituere. Oportet autem illos tres angulos rectis quatuor eſſe minores.

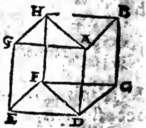




Theorema 21. Propo-
ſitio 24.

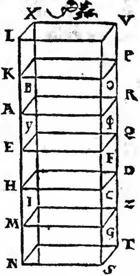


Si ſolidum ſub parallelis planis contineatur; aduerſa illius plana, ſunt parallelogramma, ſimilia, & æqualia.



Theo-

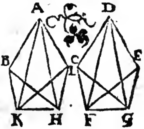
Theorema 22. Propoſitio 25.



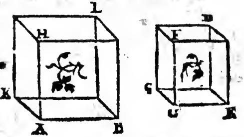
Si ſolidum parrallelopipedum, ſeu parallelis planis contentum plano ſecetur, aduerſis planis parallelo: erit quemadmodum baſis ad baſim, ita ſolidum ad ſolidum.

Problema 4. propoſitio 26.

Ad datam rectam lineã, eiuſque punctum, angulum ſolidum conſtituere, ſolido angulo dato æqualem.



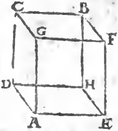
Problema 5. Propopoſitio 27.



A data recta linea, dato ſolido parallelis planis comprehenſo, ſimile, & ſimiliter poſitum ſolidũ parallelis planis contentũ deſcribere.

Theorema 23. Propoſitio 28.

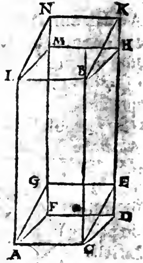
Si ſolidum parallelis planis comprehenſum ducto per aduerſorum planorum diagonios plano, ſectum ſit: illud ſolidũ ab hoc plano biſariam ſecabitur.





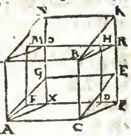
Theorema 24. Propoſitio 29.

Solida parallelapipeda, ſeu parallelis planis cõprehenſa, quæ ſuper eandem baſim, & in eadem ſunt altitudine, quorum inſiſtentes lineæ in iiſdem collocantur rectis lineis: illa ſunt inter ſe æqualia.



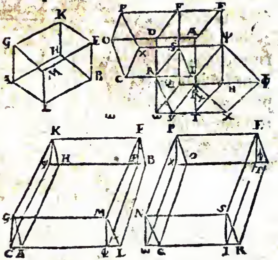
Theorema 25. Propoſitio 30.

Solida parallelis planis circumſcripta, quæ ſuper eandem baſim, & in eadem ſunt altitudine, quorum inſiſtentes lineæ non in ijſdem reperiuntur rectis lineis; illa ſunt inter ſe æqualia.



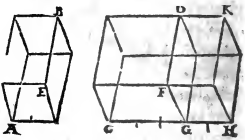
Theorema 26. Propoſitio 31.

Solida parallalelis planis, circumſcripta, quæ ſuper æqualibus baſibus, & in eadem ſunt altitudine: æqualia ſunt inter ſe.



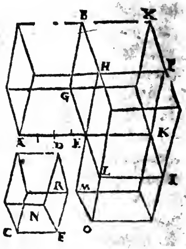
Theorema 27. Propoſitio 32.

Solida parallelis planis circumſcripta, quæ eiuſdem ſunt altitudinis; eam habent inter ſe proportioné, quam baſes.



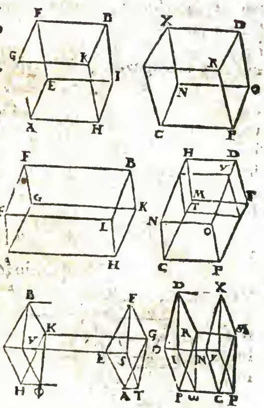
Theorema 28. Propoſitio 33.

Similia ſolida parallelis planis circumſcripta habent inter ſe proportionem homologorum laterum triplicatam.



Theo-

Theorema 29. Propoſitio 34.

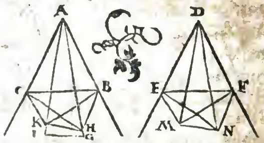


Aequalium ſolidorũ parallelis planis contentorum baſes, cum altitudinibus reciprocantur. Et ſolida parallelis planis contenta, quorum baſes cum altitudinibus reciprocantur; illa ſunt æqualia.

Theorema 30. Propoſitio 35.

Si duo plani ſint anguli æquales, quorum ver-

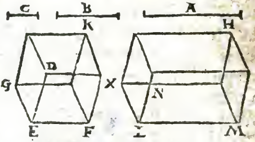
vertibus ſublimes rectæ lineæ inſiſtant, quæ cum lineis primò poſitis angulos contineant æquales, vtrunque vtrique: in ſublimibus autem lineis quælibet ſumpta ſint puncta, & ab his ad plana, in quibus conſiſtunt anguli primum poſiti, ductæ ſint perpenpendiculares; ab earum verò punctis, quæ in planis ſignata fuerint, ad angulos primum poſitos adiunctæ ſint rectæ lineæ;



hæc cum ſublimibus æquales angulos comprehendent.

Theorema 31. Propoſitio 36.

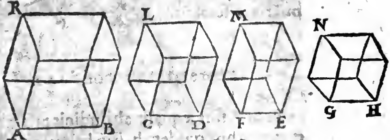
Si rectæ tres lineæ ſint proportionales;



quod ex his tribus fit ſolidum parallelis planis contentum, æquale eſt deſcripto à media linea ſolido parallelis planis comprehenſo, quod æquilaterum quidem ſit, ſed antedicto æquiangulum.

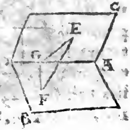
Theorema 32. Propoſitio 37.

Si rectæ quatuor lineæ ſint proportionales, illa quoque ſolida parallelis planis contenta, quæ ab ipſis lineis & ſimilia & ſimiliter diſcribuntur, proportionalia erunt. Et ſi ſolida parallelis planis comprehenſa, quæ & ſimilia & ſimiliter deſcribuntur, ſint proportionalia; illæ quoque rectæ lineæ proportionales erunt.



Theorema 33. Propoſitio 38.

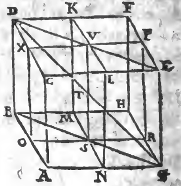
Si planum ad planum rectum ſit; & à quodam puncto eorum, quæ in vno ſunt planorum, perpendicularis ad alterum planum ducta ſit: illa, quæ ducitur perpendicularis, in communem cadet ipſorum planorum ſectionem.



Theorema 34. Propoſitio 39.

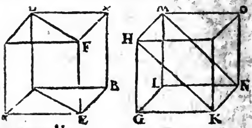
Si in ſolido parallelis planis circumſcripto, aduer-

aduerſorum planorum lateribus baſariam ſectis, educta ſint per ſectiones plana; communis illorum planorum ſectio, & ſolidi parallelis planis circumſcripti diameter, ſe mutuò biſariam ſecabũt.



Theorema 35. Propoſitio 40.

Si duo ſint æqualis altitudinis priſmata, quorum hoc quidem baſim habeat parallelogrammum; illud verò triangulum ſit autem parallelogrammũ triang.li duplum: illa priſmata erunt æqualia.



FINIS ELEMENTI XI.

EVCLI-

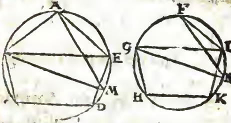
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EVCLIDIS ELEMENTVM DVODECIMVM

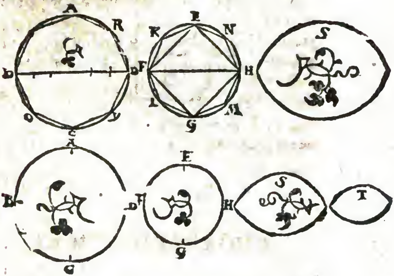
ET SOLIDORVM ſeucndum.

Theorema 1. Propoſitio 1.

Similia, quæ ſunt in circulis polygona, proportioné habent inter ſe, quã deſcripta à diametris quadrata.



Theorema 2. Propoſitio 2.

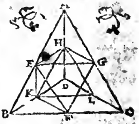


Cir-

Circuli eam inter ſe proportionem habent, quàm deſcripta à diametris quadrata.

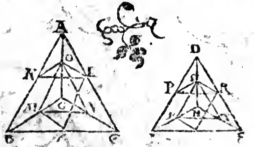
Theorema 3. Propoſitio 3.

Omnis pyramis trigonam habens baſim, in duas diuiditur pyramides non tantùm æquales & ſimiles inter ſe, ſed toti etiam pyramidi ſimiles, quarum trigonæ ſunt baſes; atque in duo priſmata æqualia, quæ duo priſmata dimidio pyramidis totius ſunt maiora.



Theorema 4. Propoſitio 4.

Si duæ eiuſdem altitudinis pyramides trigonas habeant baſes; ſit autem illarum vtraque diuiſa & in duas pyramides inter ſe æquales, totique ſimiles; & in duo priſmata æqualia; Ac eodem modo diuidatur vtraq; pyramidum, quæ ex ſuperiore diuiſione natæ ſunt; idque perpetuo fiat: quemadmodũ



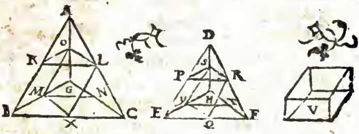
ſe habet vnius pyramidis baſis ad alterius pyramidis baſim; ita & omnia, quæ in vna

pyra-

pyramide,priſmata ad omnia,quæ in altera pyramide,priſmata multitudine æqualia.

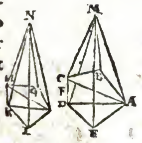
Theorema 5. Propoſitio 5.

Pyramides eiuſdem altitudinis,quarum trigonæ ſunt baſes,eam inter ſe proportionem habent,quàm ipſæ baſes.



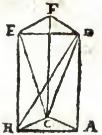
Theorema 6 Propoſitio 6.

Pyramides eiuſdem altitudinis, quarum polygonæ ſunt baſes, eam inter ſe proportionem habent quam ipſæ baſes.



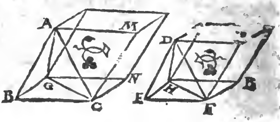
Theorema 7. Propoſitio 7.

Omne priſma trigonam habens baſim, diuiditur in tres pyramides inter ſe æquales, quarum trigonæ ſunt baſes.



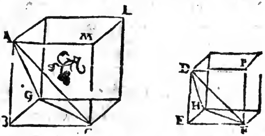
Theorema 8. Propoſitio 8.

Similes pyramides, quæ trigonas habent ba ſes in triplicata ſunt homologorum laterum proportione.



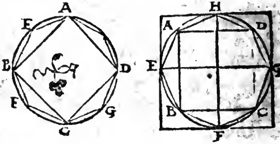
Theorema 9. Propoſitio 9.

Aequalium pyramidum, & trigonas baſes habentium, reciprocantur baſes cum altitudinibus. Et quarum pyramidum trigonas baſes habētium reciprocātur baſes cum altitudinibus; illæ ſunt æquales.



Theorema 10. Propoſitio 10.

Omnis co n' tertia pars eſt cylindri

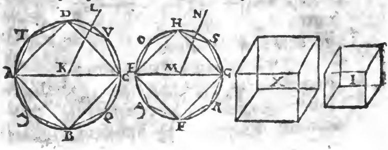


eandem

eandem cum ipſo cono baſim habentis, & altitudinem æqualem.

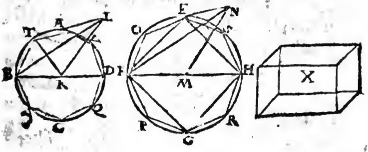
Theorema 11. Propoſitio 11.

Coni, & cylindri eiuſdem altitudinis, eam inter ſe proportionem habent, quam baſes.

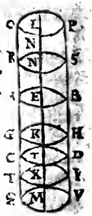


Theorema 12. Propoſitio 12.

Similes coni, & cylindri, triplicatam habẽt, inter ſe proportionem diametrorum, quæ ſunt in baſibus.



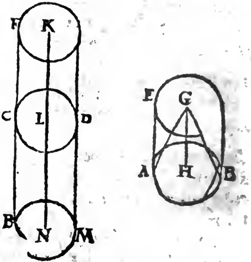
Theorema 13. Propoſitio 13.



Si cylindrus plano ſectus ſit aduerſis planis parallelo: Erit quẽadmodum cylindrus ad cylindrum, ita axis ad axem.

Theorema 14. Propoſitio 14.

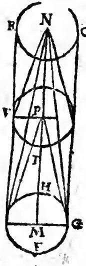
Coni, & cylindri, qui in æqualibus ſunt baſibus; eam habent inter ſe proportionẽ, quam altitudines.

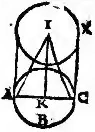


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Theorema 15. Propoſitio 15.

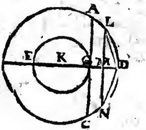
Aequalium conorum, & cylindrorum baſes cum altitudinibus reciprocā tur. Et quorum conorum & cylindrorum rum baſes cum altitudinibus reciprocā tur; illi ſunt æquales.





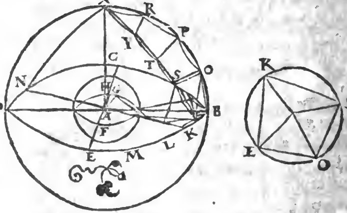
Problema 1. Propoſitio 16.

Duobus circulis circa idem centrum exiſtentibus, in maiore circulo polygonum æqualium, pariumque laterum inſcribere, quod minorem circulum non tāgat.



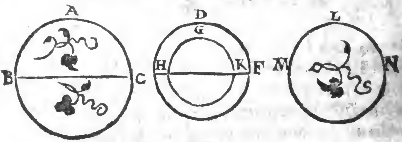
Problema 2. Propoſitio 17.

Duabus ſphæris circa idem centrum exſiſtentibus, in maiori ſphæra ſolidum polyedrum inſcribere, quod minoris ſphæræ ſuperficiem non tangat.



Theorema 16. Propoſitio 18.

Sphæræ inter ſe proportionem habent ſuarum diametrorum triplicatam.

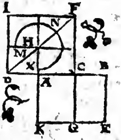


FINIS ELEMENTI XII.

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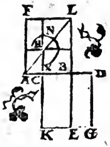
EVCLIDIS ELEMENTVM DECIMVM TERTIVM, ET SOLIDORVM tertium.

Theorema 1. Propoſitio 1.

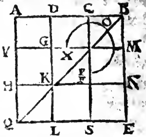


Si recta linea per extremam & mediam proportionem ſecta ſit; maius ſegmentũ, quod totius lineæ dimidium aſſumpſerit, quintuplum poteſt eius, quòd à totius dimidia deſcribitur, quadrati.

Theorema 2. Propoſitio 2.



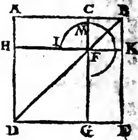
Si recta linea ſui ipſius ſegmenti quintuplum poſſit, & dupla ſegmenti huius linea per extremam & mediam proportionem ſecetur, maius ſegmentum reliqua pars eſt liniæ primùm poſitæ.



Theorema 3. Propoſitio 3.

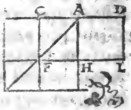
Si recta linea per extremam & mediam proportionem ſecta ſit; minus ſegmentum, quod maioris ſegmenti dimidium aſſumpſerit, quintuplum poteſt eius, quòd à maioris ſegmenti dimidio deſcribitur quadrati.

Theorema 4. Propoſitio 4.



Si recta linea per extremam & mediam proportionem ſecta ſit: quòd à tota, quodque à minore ſegmento, ſimul vtraque quadrata, tripla ſunt eius, quod à maiore ſegmento deſcribitur, quadrati.

Theorema 5. Propoſitio 5.

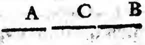


Si ad rectam lineam, quæ per extremam & mediam proportionē ſecatur, adiuncta ſit altera ſegmento maiori æqualis: tota hæc linea recta per extremam & mediam proportionem

nem ſecta; eſtque maius ſegmentum recta linea primùm poſita.

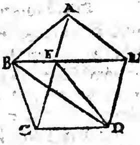
Theorema 6. Propoſitio 6.

Si recta linea ῥητὴ, ſiue rationalis, per extremam & mediam proportionem ſacta ſit; vtrunque ſegmentorum ἄλογος, ſiue irrationalis, eſt linea, quæ dicitur Reſiduum, ſeu Apotome.



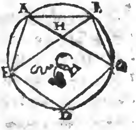
Theorema 7. Propoſitio 7.

Si pentagoni æquilateri tres ſint æquales anguli, ſiue qui deinceps, ſiue qui non deinceps ſequuntur: illud pentagonum erit æquiangulum.



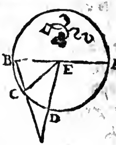
Theorema 8. Propoſitio 8.

Si pentagoni æquilateri, & æquianguli duos qui deinceps ſequuntur, angulos rectæ ſubtendāt lineæ; illę per extremam & mediam proportionem ſe mutuò ſecant; earumque maiora ſegmenta, ipſius pentagoni lateri ſunt æqualia.



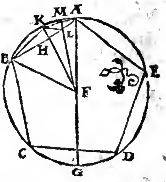
Theorema 9. Propoſitio 9.

Si latus hexagoni, & latus decagoni eidem circulo inſcriptorum compoſita ſint; tota recta linea per extremam & mediam proportionem ſecta eſt; eiuſque ſegmentum maius, eſt hexagoni latus.



Theorema 10. Propoſitio 10.

Si in circulo pentagonum æquilaterũ inſcriptum ſit; pentagoni latus poteſt & latus hexagoni, & latus decagoni, eidem circulo inſcriptorum.



Theorema 11. Propoſitio 11.

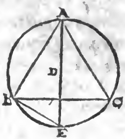
Si in circulo ῥητὴν, ſeu rationalem diametrum habente, inſcriptum ſit pentagonum æquilaterum; pentagoni latus irrationalis eſt linea, quæ vocatur minor.



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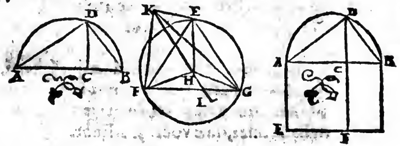
Theorema 12. Propoſitio 12.

Si in circulo inſcriptum ſit triangulum æquilaterum; huius trianguli latus potentia triplum eſt eius lineæ, quæ ex circuli centro ducitur.



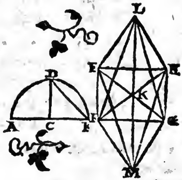
Problema 1. Propoſitio 13.

Pyramidem conſtituere; & data ſphæra cõplecti: atque docere quod illius ſphæræ diameter potentia ſeſquialtera ſit lateris ipſius pyramidis.



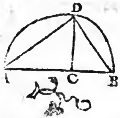
Theorema 2. Propoſitio. 14.

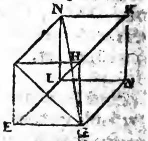
Octaedrum conſtituere, eaq; ſphę ra, qua pyramidem, complecti; atq; probarequod illius ſphærę diameter potentia dupla ſit lateris ipſius octaedri.



Problema 3. Propoſitio 15.

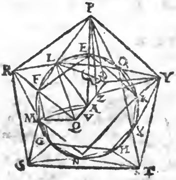
Cubum conſtituere; eaque ſphæra, qua & ſuperiores figuras complecti; atque docere quod illius ſphæræ diameter potentia tripla ſit lateris ipſius cubi.





Problema 4. Propoſitio 16.

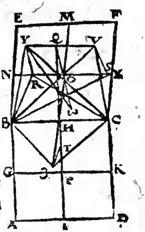
Icoſaedrum conſtituere; eademque ſphæra, qua & antedictas figuras, complecti; atque probare, quod illius Icoſaedri latus irrationalis ſit linea, quæ vocatur Minor.





Pro-

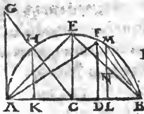
Problema 5. Propoſitio 17.

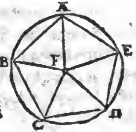


Dodecaedrum conſtituere; eademque ſphæra, qua & antedictas figuras, complecti; atq; probare quod illius dodecaedri latus irrationalis ſit linea, quæ vocatur Reſiduum, ſeu Apotome.

Problema 6. Propoſitio 18.

quinque figurarũ latera ꝓponere, & inter ſe comparare.





SCHOLIVM.

Interpretes hoc in loco demonſtrant, præter dictas quinque figuras, non poſſe aliam conſtitui figuram ſolidam, quæ ex planis & æquilateris & æquiangulis contineatur, inter ſe æqualibus.

Non

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ſieri nequit. Nec ſanè ex aliis polygonis figuris ſolidus angulus continebitur, quod hinc quoque abſurdum ſequatur.

Quamobrem perſpicuum eſt, præter dictas quinque figuras aliam figuram ſolidam non poſſe conſtitui, quæ ex planis æquilateris, & æquiangulis inter ſe æqualibus, contineatur.

Vid. Theon p. 244. Et P. Clauius p. 277.

FINIS ELEMENTI XIII.

EVCLI-

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EVCLIDIS ELEMENTVM DECIMVMQVARTVM ET SOLIDORVM QVARTVM, VT QVIdam arbitrantur; Vt alij verò, Hypſiclis Alexandrini, de quinque corporibus.

LIBER PRIMVS.

Procemium Hypſiclis Alexandrini ad protarchum.

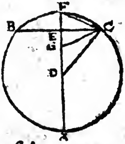
BAſilides Tyrius, Protarche, Alexandriam profectus, patriq; noſtro ob diſciplinæ ſocietatem commendatus, longiſſimo peregrinationis tempore cum eo verſatus eſt. Cũq; differerent aliquando de ſcripta ab Apollonio comparatione Dodecaedri, & Icoſaedri eidem ſphæræ inſcriptorum; quam hæc inter ſe habeant rationem, cenſuerunt ea non rectè tradidiſſe Apollonium; quæ à ſe emendata, vt de patre audire erat, literis prodiderunt. Ego autem poſtea incidi in alterum librum ab Apollonio editum, qui demonſtrationem accuratè complecteretur de re propoſita, ex eiuſq; problematis indagatione magnam equidem cœpi voluptatem. Illud certè ab omnibus perſpici poteſt, quod ſcripſit Apollonius, cum ſit in omnium manibus. Quod autem diligenti, quantum conijcere licet, ſtudio nos poſtea ſcripſiſſe videmur, id mo-

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id monumentis conſignatum tibi dedicandum duximus, vt qui feliciter cum in omnibus diſciplinis, tum vel maximè in Geometria verſatus, ſcitè ac prudenter iudices ea, quæ dicturi ſumus: Ob eam verò, quæ tibi cum patre fuit, vitæ conſuetudinem, quamq; nos complecteris, beneuolentiam, tractationem ipſam libenter audias. Sed iam tempus eſt, vt proœmium finem facientes, hanc ſytaxim aggrediamur.

Theorema 1. Propoſitio 1.

Perpendicularis lineæ, quæ ex circuli cuiuſpiam centro in latus pentagoni ipſi circulo inſcripti ducitur; dimidia eſt vtriuſque ſimul lineæ, & eius, quæ ex centro, & lateris decagoni in eodem circulo inſcripti.



Theorema 2. Propoſitio 2.

Si binæ rectæ lineæ extrema, & media proportione ſecentur; ipſæ ſimiliter ſecabuntur, in eaſdem ſcilicet proportiones.

Theorema 3. Propoſitio 3.

Si in circulo pentagonum æquilaterum inſcribatur; quod ex latere pentagoni; & quod ex ea, quæ binis lateribus pentagonis ſubtenditur, recta linea; vtraque ſimul quadrata, quintupla ſunt eius, quod ex ſemidiametro deſcribitur, quadrati.

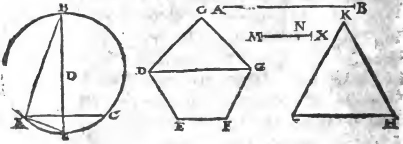
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Theorema 4. Propoſitio 4.

Si latus hexagoni alicuius circuli ſecetur extrema,& media proportione; maius illius ſegmentum erit latus decagoni eiuſdem circuli.

Theorema 5. Propoſitio 5.

Idem circulus comprehendit, & dodecaedri pentagonum, & icoſaedri triangulum, eidem ſphæræ inſcriptorum.



Theorema 6. Propoſitio 6.

Si pentagono,& æquilatero, & æquiangulo circumſcribatur circulus; ex cuius centro ad vnum pentagoni latus ducatur linea perpẽdicularis: Erit,quod ſub dicto latere,& perpendiculari continetur,rectangulum trigeſies ſumptum, dodecaedri ſuperficiei æquale.

Theorema 7. Propoſitio 7.

Si ex centro circuli triangulum icoſaedri cir-

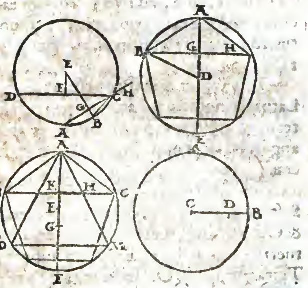
circumſcribentis, linea perpendicularis ducatur ad vnum latus trianguli: Erit, quod ſub dicto latere, & perpendiculari comprehenditur, rectangulum trigeſies ſumptum, Icoſaedri ſuperficiei æquale.

Theorema 8. Propoſitio 8.

Rectangulum contentum ſub tribus quartis partibus diametri alicuius circuli; & ſub quinque ſextis partibus lineæ ſubtendentis angulum pentagoni æquilateri in eodem circulo deſcripti; æquale eſt dicto pētagono.

Theorema 9. Propoſitio 9.

Superficies Dodecaedri ad ſuperficiem Icoſaedri, in eadem ſphæra deſcripti, eandem proportionem habet, quàm latus cubi ad latus Icoſaedri.



Theorema 14. Propoſitio 14.

Si Tetraedrum, atque octaedrum in eadem ſphæra inſcribantur: Erit baſis Tetraedri ſeſquitertia baſeos Octaedri: Superficies autem Octaedri ſeſquialtera ſuperficiei Tetraedri.

Theorema 15. Propoſitio 15.

Recta linea ex angulo quouis tetraedri in ſphæra inſcripti, per centrum ſphæræ ducta; cadit in baſeos oppoſitæ; eſtq; perpendicularis ad dictam baſin.

Theorema 16. Propoſitio 16.

Octaedrum in ſphæra inſcriptum, diuiditur in duas pyramides ęquales, & ſimiles, æqualium altitudinum, baſis vero vtriuſque pyramidis eſt quadratum ſubduplum quadrati quadrati diametri ſphæræ.

Theorema 17. Propoſitio 17.

Tetraedrum ſphæræ impoſitum ad Octaedrum in eadem ſphæra deſcriptum, ſe habet, vt rectangulum ſub linea potente viginti ſeptem ſexageſimas quartas partes quadrati lateris tetraedri; & ſub linea continente octo nonas partes eiuſdem lateris, comprehenſum, ad quadratum diametri ſphæræ.

Theorema 18. Propoſitio 18.

Linea perpendicularis ex quolibet angulo trianguli æquilateri ad baſin oppoſitam demiſſa; tripla eſt eius perpendicularis, quæ ex centro trianguli ad eandem baſin deducitur.

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Theorema 19. Propoſitio 19.

Si Octaedrum ſphæræ inſcribatur; erit ſemidiameter ſphæræ potentia tripla eius perpendicularis, quæ ex centro ſphæræ ad baſin quamcunque Octaedri ducitur.

Theorema 20. Propoſitio 20.

Duplum quadrati ex diametro cuiuſlibet ſphæræ deſcripti, æquale eſt ſuperficiei cubi in illa ſphæra collocat:perpendicularis autem à centro ſphæræ in aliquam baſin cubi demiſſa, æqualis eſt dimidio lateris cubi.

Theorema 21. Propoſitio 21.

Idem circulus comprehendit, & cubi quadratum, & Octaedri triangulum, eiuſdem ſphæræ.

Theorema 22. Propoſitio 22.

Si Octaedrum, atque Tetraedrum eidem ſphæræ inſcribantur; Erit Octaedrum ad triplum Tetraedri, vt latus Octaedri ad latus Tetraedri.

Theorema 23. Propoſitio 23.

Si recta linea propoſita potuerit totam aliquam lineam ſectam extrema, & media ratione, & maius eius ſegmentum; Item totam aliam ſimiliter ſectam, & minus eius ſegmẽtum: Erit maius ſegmentum prioris lineæ latus Icoſaedri; minus autem ſegmentum poſterioris lineæ latus Dodecaedri, eius ſphæræ cuius recta linea propoſita diameter exiſtit.

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Theorema 24. Propoſitio 24.

Si latus Octaedri potuerit maius, & minus ſegmentum rectæ lineæ extrema, ac media ratione ſectæ: poterit latus Icoſaedri in eadem ſphæra deſcripti, duplum minoris ſegmenti.

Theorema 25 Propoſitio 25.

Si recta linea diuiſa extrema ac media ratione cum minore ſegmento angulum rectũ conſtituat, cui recta ſubtendatur: Erit recta linea, quæ potentia ſit ſubdupla ipſius rectæ ſubtenſæ, latus Octaedri eius ſphæræ, in qua dictum minus ſegmentum latus exiſtit dodecaedri.

Theorema 26. Propoſitio 26.

Si latus Tetraedri poſſit maius, & minus ſegmentum lineæ rectæ extrema ac media ratione ſectæ: latus Icoſaedri eidem ſphæræ inſcripti potentia ſeſquialterum eſt minoris ſegmenti.

Theorema 27. Propoſitio 27.

Cubus ad Octaedrum in eadem cum ipſo ſphæra deſcriptum, eſt, vt ſuperficies cubi ad Octaedri ſuperficiem: Item vt latus cubi ad ſemidiametrum ſphæræ.

Theorema 28. Propoſitio 28.

Si ſint quatuor lineæ rectæ continuæ proportionalis, nec non & aliæ quatuor, ita vt ſit eadem antecedens omnium: Erit proportio tertiæ ad tertiam proportionis ſecundæ

ad ſecundam duplicata;& proportio quartę ad quartam eiuſdem proportionis ſecundæ ad ſecundam triplicata.

Theorema 29. Propoſitio 29.

Quadratum lateris trianguli æquilateri ad ipſum triangulum habet proportionem duplicatam proportionis lateris trianguli ad lineam medio loco proportionalem inter perpendicularem ab vno angulo ad latus oppoſitum ductam, & dimidium ipſius lateris.

Theorema 30. Propoſitio 30.

Si cubus, & Tetraedrum in eadem ſphæra deſcribantur; Erit quadratum cubi ad triangulum Tetraedri, vt latus Tetraedri ad lineam perpendicularem, quę ex vno angulo trianguli Tetraedri ad latus oppoſitum protrahatur.

Theorema 31. Propoſitio 31.

Latus Tetraedri potentia ſeſquialterum eſt axis, ſeu altitudinis ipſius; Axis verò, ſiue altitudo Tetraedri potentia ſeſquialtera eſt lateris cubi in eadem ſphæra deſcripti.

Theorema 32. Propoſitio 32.

Cubus triplus eſt Tetraedri eidem ſphæræ inſcripti.

FINIS ELEMENTI XIV.

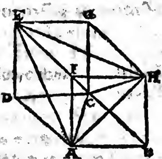
EVCLI-

EVCLIDIS ELEMENTVM DECIMVMQVINTVM ET SOLIDORVM QVINTVM, VT NONnulli putant; Vt autem alij, Hypſiclis Alexandrini, de quinque corporibus.

LIBER II.

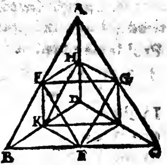
Problema 1. Propoſitio 1.

In dato cubo pyramidem (Tetraedrum inſcribere.

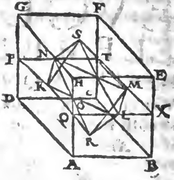


Problema 2. Propoſitio 2.

In data pyramide Octaedrum inſcribere.

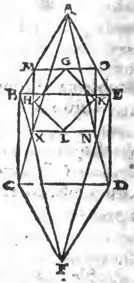


Problema 3. Propoſitio 3.



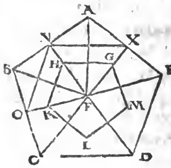
In dato cubo (hexaedro) Octaedrũ inſcribere.

Problema 4. Propoſitio 4.



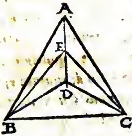
In dato octaedro cubũ inſcribere.

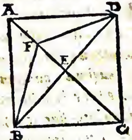
Problema 5. Propoſitio 5.

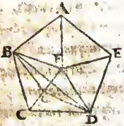


In dato Icoſaedro dodecaedrum inſcribere.

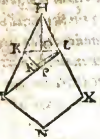
SCHOLIVM EX ZAMBERTO lib. 15. propoſ. 5. p. 260.











MEminiſſe decet, ſi quis nos roget, quot Icoſaedrum habeat latera, ita reſpondendum eſſe: Patet Icoſaedrum viginti contineri triangulis quodlibet verò triangulum rectu tribus

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conſtare lineis. Quare multiplicanda ſunt nobis viginti triangula in trianguli vnius latera ſiuntq́, ſexaginta, quorum dimidium eſt triginta. Ad eundem modum & in dodecaedro. Cum enim rurſus duodecim pentagona dodecaedrum comprehendant, itemq́, pentagonum quoduis rectis quinque conſtet lineis; quinque duodecies multiplicamus, fiunt ſexaginta, quorum rurſus dimidium eſt triginta. Sed cur dimidium capimus? Quoniam vnumquodque latus ſiue ſit trianguli, ſiue pentagoni, ſiue quadrati vt in cubo, iterato ſumitur. Similiter autem eadem via & in cubo, & in tetraedro & in octaedro latera inuenies.

Latera figurarum inuenien da.

Anguli figurarum inuenien di.

Quòd ſi item velis ſingularum quoque figurarum angulos reperire, facta eadem multiplicatione, numerum procreatum partire in numerum planorum, quæ vnum ſolidum angulum includunt: vt quoniam triangula quinque vnum Icoſaedri angulum continet, partire 60. in quinque naſcuntur duodecim anguli Icoſaedri. In dodecaedro autem tria pentagona angulum comprehendunt, partire ergo 60. in tria, & habebis dodecaedri angulos viginti. Atque ſimili ratione in reliquis figuris angulos reperies, &c.

Problema 6. Propoſitio 6.

In dato octaedro pyramidem, ſeu tetraedrum deſcribere.

pro-

Problema 7. Propoſitio 7.

In dato Dodecaedro Icoſaedrum deſcribere.

Problema 8. Propoſitio 8.

In dato Dodecaedro Cubum deſcribere.

Problema 9. Propoſitio 9.

In dato Dodecaedro Octaedrum deſcribere.

Problema 10. Propoſitio 10.

In dato Dodecaedro pyramidem deſcribere.

Problema 11. Propoſitio 11.

In dato Icoſaedro Cubum deſcribere.

Problema 12. Propoſitio 12.

In dato Icoſaedro pyramidem deſcribere.

Problema 13. Propoſitio 13.

In dato cubo Dodecaedrum deſcribere.

Problema 14. Propoſitio 14.

In dato cubo Icoſaedrum deſcribere.

Problema 15. Propoſitio 15.

In dato Icoſaedro Octaedrum deſcribere.

Problema 16. Propoſitio 16.

In dato Octaedro Icoſaedrum deſcribere.

pro-

Problema 17. propoſitio 17.

In dato Octaedro Dodecaedrum deſcribere.

Problema 18. Propoſitio 18.

In data pyramide Cubum deſcribere: hoc eſt, in propoſito Tetraedro hexaedrum delineare.

Problema 19. Propoſitio 19.

In data pyramide Icoſaedrum deſcribere.

Problema 20. Propoſitio 20.

In data pyramide Dodecaedrum deſcribere.

Problema 21. Propoſitio 21.

In dato ſolido regulari ſphęram deſcribere: hoc eſt, Interprete Campano; In fabricato quouis quinq; corporum regularium, ſphęram fabricare.

FINIS ELEMENTI XV.

EVCLI-

EVCLIDIS ELEMENTVM DECIMVM SEXTVM, ET SOLIDORVM ſextum.

Quo varia ſolidorum regularium ſibi mutuò inſcriptorum, & laterum eorundem comparationes explicantur, à Franciſco Fluſſate Candalla, & P. Chriſtophoro Clauio adiectam, & de quinque corporibus,

LIBER TERTIVS.

Theorema 1. Propoſitio 1.

SI in Dodecaedro Cubus deſcribatur, & in hoc Cubo aliud Dodecaedrum: Erit proportio Dodecaedri exterioris ad Dodecaedrum interius proportionis eius, quam habet maius ſegmentum ad minus rectæ lineę diuiſæ extrema, ac media ratione triplicata.

Theorema 2. Propoſitio 2.

Linea perpendicularis ex quouis angulo pẽtagoni æquilateri, & æquianguli in latus oppoſitum demiſſa; ſecatur à linea recta illum angulum ſubtendente, extrema ac media ratione.

Theo-

Theorema 3. Propoſitio 3.

Si ab angulis trianguli pyramidis ducantur tres lineæ rectæ, oppoſita latera ſecantes extrema ac media ratione; ita vt prope quemuis angulum ſit maius ſegmentum vnius lateris, & minus alterius: Hæ tres ſectionibus ſuis in medio producent baſin Icoſaedri in dicta pyramide deſcripti, inſcriptam quidem alij triangulo, cuius anguli latera trianguli pyramidis ſecant extrema ac media ratione; & latera ipſa bifariam ſecantur ab angulis baſis Icoſaedri.

Theorema 4. Propoſitio 4.

Minus ſegmentum lateris pyramidis extrema ac media ratione ſecti, duplum eſt potētia lateris Icoſaedri in ea pyramide deſcripti.

Theorema 5. Propoſitio 5.

Latus Cubi potentia dimidium eſt lateris pyramidis in eo deſcriptæ: Latus vero pyramidis duplum eſt longitudine lateris Octaedri ſibi inſcripti: latus denique Cubi duplum eſt potentia lateris ſibi inſcripti Octaedri.

Theorema 6. Propoſitio 6.

Latus Dodecaedri maius ſegmentum eſt rectæ lineæ quæ potentia eſt dimidia lateris pyramidis ſibi inſcriptæ.

Theorema 7. Propoſitio 7.

Si in Cubo deſcribatur & Icoſaedrum, & Dode-

Dodecaedrũ;Latus Icoſaedri mediũ propor tionale erit inter latus Cubi,&Dodecaedri.

Theorema 8. Propoſitio 8.

Latus pyramidis potentia Octodecuplum eſt lateris Cubi in ea deſcripti.

Theorema 9. Propoſitio 9.

Latus pyramidis potentia Octaedecuplum eſt rectæ lineę extrema ac media ratione ſectæ,cuius maius ſegmentum eſt latus Dodecaedri in pyramide deſcripti.

Theorema 10. Propoſitio 10.

Si in Octaedro Icoſaedrum deſcribatur: Erit latus Icoſaedri potentia duplum minoris ſegmenti lateris Octaedri extrema ac media ratione diuiſi.

Theorema 11. Propoſitio 11.

Latus Octaedri potentia quadruplum eſt ſeſquialterum lateris Cubi in ipſo deſcripti.

Theorema 12. Propoſitio 12.

Latus Icoſaedri maius ſegmentum eſt eius rectæ lineæ extrema ac media ratione ſectę, quæ potentia dupla eſt lateris Octaedri in eo Icoſaedro deſcripti.

Theorema 13 Propoſitio 13.

Latus Cubi ad latus Dodecaedri in ipſo deſcripti proportionem habet duplicatam eius,quam habet maius ſegmentum ad min', rectę lineędiuiſę extrema ac media ratione. Latus vero Dodecaedri ad latus cubi in ipſo deſcri-

deſcripti, proportionem habet, quam minus ſegmentum ad maius, eiuſdem rectæ lineæ.

Theorema 14. Propoſitio 14.

Latus octaedri ſeſquialterum eſt lateris pyramidis ſibi inſcriptæ.

Theorema 15. Propoſitio 15.

Si ex quadrato diametri Icoſaedri auferatur triplum quadrati lateris cubi in eo deſcripti; relinquitur quadratum ſeſquitertium quadrati lateris Icoſaedri.

Theorema 16. Propoſitio 16.

Latus Dodecaedri minus ſegmentum eſt rectæ lineæ extrema ac media ratione diuiſæ, quę duplum poteſt lateris octaedri in eo deſcripti.

Theorema 17. Propoſitio 17.

Diameter Icoſaedri poteſt, & ſui ipſius lateris ſeſquitertium, & lateris pyramidis in eo deſcriptæ ſeſquialterum.

Theorema 18. Propoſitio 18.

Latus Dodecaedri ad Icoſaedri ſibi inſcripti latus, ſe habet, vt minus ſegmentum lineæ perpendicularis ab vno angulo pentagoni ad latus oppoſitum ductæ, atque extrema ac media ratione diuiſæ, ad partem eiuſdem lineæ inter centrum pentagoni, & latus eiuſdem poſitæ.

Problema 19. Propoſitio 19.

Si dimidium lateris Icoſaedri extrema ac

media

media ratione ſectum fuerit, minuſque eius ſegmentum à toto latere Icoſaedri ſublatum; à reliqua quoque recta linea pars rurſum tertia detracta: Relinquetur latus Dodecaedri in Icoſaedro deſcripti.

Theorema 20 Propoſitio. 20.

Cubus ſibi inſcriptæ pyramidis triplus eſt.

Theorema 21. Propoſitio 21.

Pyramis ſibi inſcripti Octaedri dupla eſt.

Theorema 22. Propoſitio 22.

Cubus ſibi inſcripti Octaedri ſextuplus eſt.

Theorema 23. Propoſitio 23.

Octaedrum ſibi inſcripti Cubi quadruplum ſeſquialterum eſt.

Theorema 24. Propoſitio 24.

Octaedrum ſibi inſcriptæ pyramidis tredecuplum ſeſquialterum eſt.

Theorema 25. Propoſitio 25.

Pyramis ſibi inſcripti Cubi noncupla eſt.

Theorema 26. Propoſitio 26.

Octaedrum ad Icoſaedrum ſibi inſcriptum proportionem habet, quam duæ baſes octaedri ad quinque baſes Icoſaedri.

Theorema 27. Propoſitio 27.

Icoſaedrum ad Dodecaedrum ſibi inſcriptum, proportionem habet compoſitam ex proportione lateris Icoſaedri ad latus cubi in eadem cum Icoſaedro ſphæra deſcriſcripti; & ex proportione triplicata eius quam habet diameter Icoſaedri ad rectam

lineam contra baſium Icoſaedri oppoſitarum coniungentem.

Theorema 28. Propoſitio 28.

Dodecaedrum excedit Cubum ſibi inſcriptum parallelepipedo, cuius quidem baſis à quadrato Cubi deficit rectangulo contento ſub latere Cubi, tertiaque parte, minoris ſegmenti eiuſdem lateris Cubi: At verò altitudo ab altitudine, ſiue latere Cubi deficit, minore ſegmento eius lineæ, quæ dimidiati lateris Cubi ſegmentum minus exiſtit.

Theorema 29. Propoſitio 29.

Dodecaedrum ad Icoſaedrum ſibi inſcriptum, proportionem habet compoſitam ex proportione triplicata eius, quam habet diameter Dodecaedri ad rectam lineam contra baſium Dodecaedri oppoſitarum copulantem; & ex proportione lateris Bubi ad latus Icoſaedri in eadem ſphæra cum Cubo deſcripti.

Theorema 30. Propoſitio 30.

Dodecaedrum pyramidis, in qua deſcribitur, duas nonas partes continet, minus duobus parallelepipedis; quorum vnius longitudo lateri Cubi in eadem pyramide deſcripti, æqualis eſt; latitudo verò tertiæ parti mi-

ti minoris ſegmenti lateris eiuſdem Cubi; altitudo denique à latere eiuſdem Cubi deficit minore ſegmento eius lineæ, quæ dimidiati lateris Cubi eiuſdem minus ſegmẽtum exiſtit: Alterius autem & longitudo,& latitudo lateri Cubi prædicti, eſt æqualis; altitudo vero minus ſegmentum eius lineæ quæ dimidiati lateris Cubi eiuſdem ſegmentum minus exiſtit;ita vt amborum altitudines ſimul altitudini,ſiue lateri eiuſdem Cubi ſint æquales.

Theorema 31. Propoſitio 31.

Octaedrum excedit ſibi inſcriptum Icoſaedrum parallelepipedo, cuius baſis eſt quadratum lateris Icoſaedri; altitudo vero maius ſegmentum ſemidiametri Octaadri extrema ac media ratione ſectæ.

DE QVINQVE CORPORVM regularium deſcriptione in data ſphæra, ex Pappo Alexandrino.

Lemma I.

Datis duobus circulis in ſphæra parallelis, dataque in vno eorum linea recta;ducare in altero diametrum huius rectæ datæ parallelam.

Lemma II.

Si in planis parallelis deſcripti ſint duo circuli, in quibus duæ rectæ parallelæ abſcindant arcus ſimiles: Erunt duæ rectæ coniungentes extrema vnius rectæ cum centro parallelæ duabus rectis, quæ extrema alterius rectæ cum centro coniungant.

Lemma III.

Si in ſphæra ſint duo circuli paralleli, & æquales, in quibus ductæ ſint duæ rectæ parallelæ, & æquales, ad eaſdem partes centrorum: Rectæ harum parallelarum extrema puncta ad eaſdem partes coniungentes, æquales quoque, & parallelæ ſunt, & ad plana circulorum perpendiculares.

Lemma IV.

Si in ſphæra ſint duo circuli paralleli, & æquales, in quibus ductæ ſint duæ rectæ parallelæ, & æquales, non ad eaſdem partes cētrorum: Rectæ lineæ harum parallelarum extrema puncta non ad eaſdem partes coniungentes, in centro ſphæræ ſeſe interſecant; ac proinde diametri ſphæræ erunt, & inter ſe æquales: Rectæ verò lineæ earundem parallelarum extrema puncta ad eaſdem partes connectentes, & æquales, & parallelæ inter ſe

ter ſe ſunt, & cum parallelis rectos angulos conſtituunt.

Lemma V.

Si in ſphæra ſint duæ rectæ parallelæ; rectæ earum puncta extrema ad eaſdem partes coniungentes, æquales inter ſe erunt: Et ſi parallelæ ſint æquales, coniungentes non ſolùm æquales, ſed & parallelæ erunt, rectoſque cum ipſis angulos conficient.

Lemma VI.

In data ſphæra duos circulos æquales, ac parallelos deſcribere; ita vt diameter ſphæræ ſit vtriuſque diametri potentia ſeſquialtera.

Problema 1. Propoſitio 1.

In data ſphæra pyramidem trigonam deſcribere.

Problema 2. Propoſitio 2.

In data ſphæra Octaedrum deſcribere.

Problema 3. Propoſitio 3.

In data ſphæra Cubum deſcribere.

Problema 4. propoſitio 4.

In data ſphæra Icoſaedrum deſcribere.

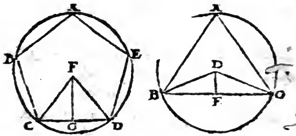
Problema 5 Propoſitio 5.

In data ſphæra Dodecaedrum deſcribere.

SCHOLIVM EX P. CLAVIO.

EX his, quæ hoc in loco, & lib. 13. & lib. 14. demonſtrata ſunt, facile etiam oſtenditur, ſi omnia quinque corpora regularia in vna eademque ſphæra deſcribantur, maximum omnium eſſe Dodecaëdrum: Deinde Icoſaëdrum maius reliquis tribus: Tertio Cubum maiorem reliquis duobus: Octaëdrum denique Tetraëdro eſſe maius. Ex quo euidenter conſtabit, Euclidem recto ordine quinque hæc corpora conſtruxiſſe, cum poſt Tetraëdrum ſtatim Octaëdrum non autem Cubum conſtituerit. Ita enim à minoribus ad maiora progreſſus eſt.

FINIS ELEMENTI XVI. ET Vltimi.



Pagina 208. Theoremate 6. deſunt hæ duæ Figuræ.

ERRATA SIC CORRIGATO.

Pagina 1. verſu 5 legatur: Punctum eſt, cuius pars nulla eſt. *Punctum*

p.3.v.10.vel pro vt.q. 5. v. 6. comprehenduntur pro comprehenditur. p. 8. v.6. perallelis pro perallelis, Ibid.dicuntur pro dicentur p.9.v. 5. poſtponantur. A B.C.D.v.penult: poſt ſi addatur ab p.13.19.ad pro ab p. 17.v.14. Propoſit: 18. p. 18.v.19. maiorem pro minorem.p 34. v.12.ipſas pro ipſis.p.40.v.16.circu lus pro circulum.p.41.v.20.prob.2. pro prob. 12. p. 50.v.12.figuram profiguræ.p.68.v. 2. diuidendo diuidando.Ibid.v.18. priorum pro priorem.p.71.poſt proport.add: ex.Ibid. v.15. exceedens pro excadens. p.76.v.15.æqualia pro æquales p. 77.v.1.poſt Et addat. ſi. Ibid.v.18.Problema proTheorema p.85.v.6. vnũquodque.p.88.vnius numeri p.90.v.2.& ille p. 93.v.13.& reliquus ad p.101. v. 4. dele definitiones p. 116. v.Problema 2.p.121.v. 5. primæ nempe mag nitudinum p.126.v.penu. Problema 3.p.138.dele eſt p.146.v.10.dele lineam p.156.v.20. Reperire primũ reſiduum adde ſeu apotomen p.164.v.11.dele ſeu p. 169.v.20.ad rectos.p. 170.v.23.pluriũ.p.173. v. 3. aduerſis Ibid.9. Cubus ſeu hexaedrum. p. 181. v.4.ſub parallelis.p.186 v.1.verticibus.p.189:v.5.ſecundum. p.197.v.11.dele,rum.Ibid.v.vlt.circulũ non tangat. p.190 v.1.pro ſecta,lege ſecatur.Ibid. v.5. ſecta ſit,p. 201.v.Problema 2.p.204.v. 9. ſex autem triangulis & ęquilateris.p. 297.v. 8. vt proœmio finem. pag. 210. verſ.26. Si ſpæræ duo. pa. 211. verſ. 10. cadit in centum baſeos ibidem verſ.17.dele quadrati.

Cætere beneuolus lector per ſe facile corriget.

FINIS.

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